Online Supplement 4: Tutorial on the discriminant validity calculations

This document provides a tutorial on the calculation of the various discriminant validity statistics presented in the article. We start by explaining the manual calculations, following by a detailed explanation on how to estimate the required models using AMOS, LISREL, Mplus, R and Stata.

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# Overview

Table 1 shows an example correlation matrix that we will use throughout the tutorial. The matrix is color-coded to highlight the relevant covariances used in the calculations. Because some statistical software support the discriminant validity analyses only using raw data, we will later provide SPSS, Stata, and R code for generating a dataset matching this covariance matric. The data below (SPSS and txt version) and the source codes are available from <https://github.com/eunscho/DiscriminantValidityTutorial>. We also provide SPSS, Stata, and R code for generating this matrix for analysis in SEM software.

Table 1 Example covariance matrix

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ACSI1 | ACSI2 | ACSI3 | CUEX1 | CUEX2 | CUEX3 | PERQ1 | PERQ2 | PERQ3 | PERV1 | PERV2 |
| ACSI1 | 4.00 |  |  |  |  |  |  |  |  |  |  |
| ACSI2 | 3.23 | 4.41 |  |  |  |  |  |  |  |  |  |
| ACSI3 | 2.66 | 2.67 | 3.61 |  |  |  |  |  |  |  |  |
| CUEX1 | 1.81 | 1.50 | 1.56 | 4.41 |  |  |  |  |  |  |  |
| CUEX2 | 1.85 | 1.62 | 1.63 | 2.63 | 4.84 |  |  |  |  |  |  |
| CUEX3 | 1.24 | 1.16 | 1.09 | 1.55 | 1.72 | 5.29 |  |  |  |  |  |
| PERQ1 | 3.04 | 2.83 | 2.35 | 2.07 | 1.96 | 1.31 | 3.61 |  |  |  |  |
| PERQ2 | 2.81 | 2.57 | 2.19 | 1.55 | 1.74 | 1.12 | 2.67 | 3.24 |  |  |  |
| PERQ3 | 1.73 | 1.64 | 1.32 | 0.89 | 1.05 | 1.41 | 1.62 | 1.62 | 2.89 |  |  |
| PERV1 | 2.66 | 2.49 | 2.12 | 1.40 | 1.43 | 0.95 | 2.22 | 2.01 | 1.25 | 3.24 |  |
| PERV2 | 2.99 | 2.86 | 2.42 | 1.52 | 1.50 | 1.01 | 2.34 | 2.14 | 1.31 | 3.05 | 4.84 |

Note: Based on Table 5 as given by Henseler, Ringle, and Sarstedt (2015), converted into a covariance matrix. The standard deviation values used in the conversion are arbitrary numbers, from ACSI1 to PERV2 in the following order: (2, 2.1, 1.9, 2.1, 2.2, 2.3, 1.9, 1.8, 1.7, 1.8, 2.2).

# Techniques that do not require SEM software

## Scale score correlation

### How often is it used?

AMJ 25.9%, JAP 11.0%, ORM 55.0%

This is probably the most easily understood among the various techniques, so it is commonly used.

### How to obtain

Scales scores are typically the unweighted sum or mean of observed item scores.

ACSI = ACSI1 + ACSI2 + ACSI3

CUEX = CUEX1 + CUEX2 + CUEX3

PERQ = PERQ1 + PERQ2 + PERQ3

PERV = PERV1 + PERV2

The scale score correlation can be calculated using the calculation rules for covariances. (See e.g. <https://en.wikipedia.org/wiki/Covariance#Covariance_of_linear_combinations>)

Repeat the calculation as above to get the following scale score correlation table. This is the most common type of correlation matrix in published articles.

Table 2 Scale score correlation matrix

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .485 | 1.000 |  |  |
| PERQ | .817 | .550 | 1.000 |  |
| PERV | .765 | .404 | .645 | 1.000 |

### Problems / Limitations

Scale score correlation is a function of 'true correlation' and scale score reliability. That is, a low scale score correlation may be due to a low true correlation, or a low reliability. Therefore, these correlations alone do not allow any conclusions about discriminant validity to be drawn.

## Disattenuated correlation using tau-equivalent reliability

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

As mentioned above, scale score correlation is a function of 'true correlation' and scale score reliability. Therefore, the true correlation can be estimated by adjusting the scale score correlation to account for its unreliability. The estimate of true correlation obtained in this way is called disattenuated correlation. The effectiveness of this technique depends on a) how accurate the reliability estimates are used and b) to the extent that the indicators are contaminated by non-random measurement error. In typical applications, non-random measurement error is assumed to not exist.

There are many ways to estimate scale score reliability, but the most commonly used is tau-equivalent reliability. This reliability coefficient is often incorrectly known as Cronbach's alpha. Kuder and Richardson (1937), not Cronbach (1951), developed this coefficient first (see Cho & Kim, 2015). Its formula is as follows.

For example, applying this formula to ACSI yields the following value:

Repeat the above calculations to obtain the following reliability estimates (statistical software such as SPSS provides automated calculations).

Table 3 Tau-equivalent reliabilites

|  |  |
| --- | --- |
|  |  |
| ACSI | .882 |
| CUEX | .672 |
| PERQ | .822 |
| PERV | .860 |

After the reliability estimates have been calculated, disattenuated correlationcan be calculated as follows:

For example, the disattenuated correlation of ACSI and CUEX can be obtained as follows using the previously presented results:

Repeat the calculation as above to get the following disattenuated correlation table.

Table 4 Correlations disattenuated with tau-equivalent reliability coefficients

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .631 | 1.000 |  |  |
| PERQ | .960 | .740 | 1.000 |  |
| PERV | .879 | .532 | .767 | 1.000 |

### Problems / Limitations

Tau-equivalent reliability is widely misunderstood and misused (Cho, 2016; Cho & Kim, 2015). Perhaps the most common misconception is that it is a general reliability coefficient that can be applied to all data. Tau-equivalent reliability is based on the assumption that each variable is influenced by the true score to the same extent, which implies equal covariances as shown in Table 5. For example, to satisfy tau-equivalence, the covariances between ACSI1, ACSI2, and ACSI3 that make up ACSI must all have the same value. The covariance of ACSI1-ACSI3 is 2.66 and the covariance of ACSI2-ACSI3 is 2.67, so it appears that tau-equivalence is met. However, the covariance of ACSI1-ACSI2 is 3.23, which differs greatly from the other two values and thus does not meet tau-equivalence. Therefore, a technique that does not depend on the assumption of tau-equivalence is needed.

Table 5 Examples of parallel, tau-equivalent, and congeneric covariances

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A. Parallel data | | | | B. Tau-equivalent data | | | | C. Congeneric data | | | |
|  | X1 | X2 | X3 |  | X1 | X2 | X3 |  | X1 | X2 | X3 |
| X1 | 10 | 4 | 4 | X1 | 10 | 4 | 4 | X1 | 10 | 2 | 3 |
| X2 | 4 | 10 | 4 | X2 | 4 | 9 | 4 | X2 | 2 | 9 | 6 |
| X3 | 4 | 4 | 10 | X3 | 4 | 4 | 11 | X3 | 3 | 6 | 11 |
| All three values are correct | | | | and are correct | | | | Only is correct  , | | | |

## Disattenuated correlation using parallel reliability (so called HTMT)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

This technique is rarely used by organizational researchers yet. However, its popularity has exploded in other related fields after Henseler, Ringle and Sarstedt (2015) has introduced this technique under the name Heterotrait-Monotrait (HTMT).

### How to obtain

Parallel reliability is often mistakenly referred to as standardized alpha, which gives a misleading impression that it is a variant of tau-equivalent reliability (i.e., alpha). The two reliability coefficients are based on different assumptions and are independent formulas. As with tau-equivalent reliability, parallel reliability is also readily available in popular statistical software packages such as SPSS. Its formula is as follows.

The symbol refers to the mean correlation between the items.

These correlations between items can be obtained using the correlation formula presented above. For example, the correlation of ACSI1-ACSI2,

The correlations of ACSI1-ACSI3 and ACSI2-ACSI3 are .700 and .669, respectively. Substituting these values into the above formula:

Repeat the above calculations to obtain the following reliability estimates.

Table 6 Parallel reliabilities

|  |  |
| --- | --- |
|  |  |
| ACSI | .882 |
| CUEX | .676 |
| PERQ | .820 |
| PERV | .870 |

The disattenuated correlation of ACSI and CUEX can be obtained as follows.

Repeat the calculation as above to get the following disattenuated correlation table.

Table 7 Correlations disattenuated with parallel reliability coefficients

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .633 | 1.000 |  |  |
| PERQ | .955 | .737 | 1.000 |  |
| PERV | .877 | .534 | .766 | 1.000 |

### Problems / Limitations

Parallel reliability is less accurate than tau-equivalent reliability. Therefore, disattenuated correlation using parallel reliability is less effective than disattenuated correlation using tau-equivalent reliability for assessing discriminant validity. In addition to tau-equivalence, parallel reliability also requires the assumption that the variance of each item is identical (i.e., the condition of being parallel). In other words, it is more difficult to satisfy the assumption of parallel reliability than to satisfy that of tau-equivalent reliability. Parallel reliability has no computational advantage over tau-equivalent reliability. Therefore, the use of this technique for the assessment of discriminant validity is difficult to justify.

## Disattenuated correlation using congeneric reliability

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

Congeneric reliability is more accurate than tau-equivalent reliability but less frequently used. This coefficient is often referred to as composite reliability or McDonald's omega. Jöreskog (1971) developed this coefficient first and named its measurement model a congeneric model. The likely reason for the lower adoption of this coefficient compared to the previously presented reliability coefficients is that this coefficient cannot be calculated directly from the data, but requires the estimation of a factor analysis model, typically a CFA. However, most SEM software, except EQS, does not automatically provide reliability estimates, which means that this index needs to be calculated manually. Although congeneric reliability is based on SEM, Cho (2016) has developed RelCalc, a program that estimates congeneric reliability using only Microsoft Excel without SEM software. The following are estimates of congeneric reliability obtained through RelCalc.

After standardized factor loadings () and error variances () have been estimated, congeneric reliability can be calculated using the following formula:

We apply this equation to SEM estimates that are reported later in the tutorial:

Applying the formula gives the following reliability estimates:

Table 8 Congeneric reliabilities

|  |  |
| --- | --- |
|  |  |
| ACSI | .885 |
| CUEX | .687 |
| PERQ | .842 |
| PERV | .860 |

The underlying disattenuation equation is the same as earlier.

Table 9 Disattenuated correlations using congeneric reliability

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .622 | 1.000 |  |  |
| PERQ | .947 | .723 | 1.000 |  |
| PERV | .877 | .526 | .758 | 1.000 |

### Problems / Limitations

Disattenuated correlation using congeneric reliability is more general than the previous methods. Generally, the disattenuatio techniques are commonly recommended as alternatives for researchers who have difficulty using SEM software. However, disattenuated correlation technique itself has problems. First, a correlation coefficient that is greater than 1 or less than -1 may be derived. Second, the correlation estimates can be less precise and the calculation process has more steps than when directly estimating factor correlations based on CFA or SEM thus leaving more room for errors. Third, although it is possible to obtain the confidence interval of disattenuated correlation, doing this correctly using equiations is complicated. Another alternative is to apply boostrapping, but this often requires a bit of programming of the statistical software and can be too computationally intensive to be a practical alternative.

## Cross-loadings (obtained from exploratory factor analysis)

### How often is it used?

AMJ 3.7%, JAP 0%, ORM 15.0%

### How to obtain

The inspection of cross-loadings is sometimes suggested as a way for assessing discrimination validity. However, the term cross-loading has at least two different meanings in the literature that need to be explicitly explained.

1. What is a loading? That is, is it a pattern coefficient or a structure coefficient?
2. What is a *cross*-loading? That is, does the determination of the existence of a problematic cross-loadings require absolute comparisons (e.g., cutoff point) or relative comparisons (e.g., other coefficient value)?

The proper application of an exploratory factor analysis (EFA) is a complex subject in itself, so we provide just a brief demonstration instead of fully explaining the analysis. We selected maximum likelihood as the factor extraction method, Varimax as the orthogonal rotation, and Promax as the oblique rotation. The number of factors was determined to be two when determined according to the commonly used criterion that the eigenvalues of the factors should all be greater than one[[1]](#footnote-1).

First, let us consider the first question. Factor loadings represent pattern coefficients or structure coefficients depending on the context. The pattern coefficients indicate how the item value changes when the factor’s (unobserved) value changes by one unit holding other factors constant, which is similar to the coefficient in the regression analysis. The structure coefficient is the correlation between items and factors.

Exploratory factor analysis results will generally need to be rotated to make them more interpretable. We start by considering the orthogonal rotation. The word orthogonal is a geometric term, and its corresponding statistical term is ‘being uncorrelated with each other’. That is, the correlation between the two factors is fixed as follows.

Table 10 Factor correlation matrix after orthogonal rotation

|  | Factor 1 | Factor 2 |
| --- | --- | --- |
| Factor 1 | 1 | 0 |
| Factor 2 | 0 | 1 |

The factor structure matrix is the matrix product of the factor pattern matrix and the factor correlation matrix. Because the correlation matrix is an identity matrix (i.e., a matrix composed of diagonal elements of 1 and non-diagonal elements of 0) in orthogonal rotation, the structure matrix and the pattern matrix are identical. The following is the result of Varimax rotation of the example.

Table 11 Rotated pattern/structure matrix

|  | Factor 1 | Factor 2 |
| --- | --- | --- |
| ACSI1 | .675 | .616 |
| ACSI2 | .588 | .566 |
| ACSI3 | .553 | .528 |
| CUEX1 | .496 | .206 |
| CUEX2 | .500 | .191 |
| CUEX3 | .339 | .114 |
| PERQ1 | .793 | .426 |
| PERQ2 | .754 | .423 |
| PERQ3 | .515 | .267 |
| PERV1 | .380 | .793 |
| PERV2 | .249 | .845 |

While this example would suggest that there is not much difference between the pattern and structure matrices, it would be a mistake to assume so. Orthogonal rotation is based on an unrealistic assumption that factors are uncorrelated and should thus be avoided in research that aims to study correlations between constructs. The following is the result of Promax rotation of the example.

Table 12 Structure matrix and pattern matrix after oblique rotation

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Structure matrix | |  | Pattern matrix | |  | Correlation matrix | |
| Factor 1 | Factor 2 | Factor 1 | Factor 2 | Factor 1 | Factor 2 |
| ACSI1 | **.876** | .848 | **.549** | .417 | 1.000 | .785 |
| ACSI2 | **.777** | .766 | **.458** | .406 | .785 | 1.000 |
| ACSI3 | **.728** | .716 | **.433** | .376 |  | |
| CUEX1 | **.536** | .401 | **.576** | -.051 |
| CUEX2 | **.533** | .389 | **.592** | -.076 |
| CUEX3 | **.355** | .250 | **.413** | -.074 |
| PERQ1 | **.899** | .729 | **.853** | .059 |
| PERQ2 | **.864** | .709 | **.799** | .082 |
| PERQ3 | **.580** | .465 | **.561** | .024 |
| PERV1 | .688 | **.879** | -.005 | **.883** |
| PERV2 | .594 | **.869** | -.231 | **1.050** |

For example, the structure coefficient between ACSI and Factor 1 can be calculated as follows.

When oblique rotation is used, the meaning of the term “factor loading” is ambiguous as it can refer to either structure coefficients or pattern coefficients, which are clearly different quantities as the example shows. It is also worth noting that in the above correlation matrix, the correlation between the two factors is very high at .785. That is, the assumption that the two factors are not correlated with each other is far from reality and the results of orthogonal rotation can be very misleading.

Now, let's consider the second question. In other words, what value should the loading be greater than to be considered a problematic 'cross-loading'? The first method to detect if cross-loading exists is absolute comparison. If the absolute value of the coefficient between an item and a factor is higher than an arbitrary cutoff point (e.g., .3, .4, .5), then the item is considered to be 'loaded' on the factor. If an item is 'loaded' on more than one factor, the item is considered to be 'cross-loaded'. For example, let's say the cutoff point is .4, which is a fairly commonly used cutoff. In the above structure matrix, all items are cross-loaded except for CUEX2 and CUEX3. CUEX3 is not loaded on any factor. Cross-loadings are less common in the pattern matrix and are observed only in ACSI1 and ACSI2. The problem with this method is that the choice of an cutoff is arbitrary and changing the cutoff changes the judgment of cross-loadings substantially. For example, if the cutoff point is .5, there is no cross-loading in the above pattern matrix, but ACSI2, ACSI3, and CUEX3 are not loaded on any factor.

The second method is relative comparison. In this comparison, an item is 'loaded' on a factor which it has highest loading on among all the factors. These coefficient values are shown in boldface in the table above. We can think of two rules related to cross-loadings. The first rule is what we call row comparison (Henseler et al., 2015). That is, 'for *an item*, if the absolute value of the coefficient between the item and the loaded factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between the item and any unloaded factors (e.g., ACSI-Factor2), the item is cross-loaded. ' However, according to the above definition the loaded coefficient value is the maximum value among the coefficient values of the same row, so that a cross-loading by this definition cannot occur. If you look at the table above, you can confirm that there is no cross-loading by this rule at all. Another variant of this rule is that the loading must be at least .2 or some other arbitrary number higher than any of the potential cross-loadings. According to this rule, for example all the ACSI items would cross-load on both factors regardless whether pattern or structure coefficients are inspected.

The second rule is what we call column comparison (Thompson, 1997). That is, 'for *a factor*, if the absolute value of the coefficient between the loaded item and the factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between any unloaded items and the factor (e.g., PERV1-Factor1), the item is cross-loaded. ' Even with this rule, there is no cross-loading in the above pattern matrix. However, in the structure matrix there are cross-loadings in CUEX1, CUEX2, CUEX3, and PERQ3. For example, the coefficient of CUEX1-Facor1 is .536, of which absolute value is smaller than that of the coefficient of .688 of PERV1-Factor1.

### Problems / Limitations

One disadvantage of this technique is the lack of consensus on exactly what cross-loadings are. More fundamentally, examining cross-loadings to assess discriminant validity does not fit the definition of discriminant validity. For further discussion, see the main text.

## Factor correlation (obtained from EFA)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

As each variable name implies, our example is expected to include four factors. However, as a result of the usual procedure for determining the number of factors, it was determined to be two, which was used for simplicity in the previous example. One possible alternative is to analyze the number of factors fixed at four. The following table shows the results.

Table 13 Factor correlations of a four factor solution

| Factor correlation matrix | | | | |
| --- | --- | --- | --- | --- |
|  | Factor 1 | Factor 2 | Factor 3 | Factor 4 |
| Factor 1 | 1.000 | .741 | .550 | .549 |
| Factor 2 | .741 | 1.000 | .420 | .348 |
| Factor 3 | .550 | .420 | 1.000 | .444 |
| Factor 4 | .549 | .348 | .444 | 1.000 |

### Problems / Limitations

EFA is a technique of 'letting the data speak', and the theoretical background and the intention of the researcher are not taken into consideration at all. Therefore, EFA often produces results that are difficult to interpret. The following shows the derived pattern matrix. Many items are 'loaded' on unintended factors. This discussion shows why CFA, not EFA, should be used when looking for factor correlation.

Table 14 Rotated factor pattern matrix

|  | Factor 1 | Factor 2 | Factor 3 | Factor 4 |
| --- | --- | --- | --- | --- |
| ACSI1 | **.770** | .212 | -.006 | -.027 |
| ACSI2 | **.722** | .201 | -.076 | -.028 |
| ACSI3 | **.556** | .223 | .063 | -.016 |
| CUEX1 | .001 | .014 | **.844** | -.009 |
| CUEX2 | .101 | -.010 | **.574** | .119 |
| CUEX3 | -.135 | .043 | .185 | **.573** |
| PERQ1 | **.878** | -.078 | .172 | -.049 |
| PERQ2 | **.911** | -.081 | .008 | .026 |
| PERQ3 | .427 | -.031 | -.151 | **.485** |
| PERV1 | .209 | **.695** | .020 | .012 |
| PERV2 | -.022 | **.907** | -.004 | .019 |

We will now describe techniques for evaluating discriminant validity using SEM software. Numerous SEM software packages are being used now. The next section describes four software applications, AMOS, LISREL, MPLUS, R, and Stata (in alphabetical order).

# Techniques that require SEM software: AMOS version

Input data are provided as ACSICovData.sav and the AMOS Graphics file is ACSITutorial.amw in <https://github.com/eunscho/DiscriminantValidityTutorial>. The input data can also be created by running the following syntax in SPSS.

DATA LIST LIST (",")/ ROWTYPE\_ (A3) VARNAME\_ (A6) ACSI1 ACSI2 ACSI3 CUEX1 CUEX2 CUEX3 PERQ1 PERQ2 PERQ3 PERV1 PERV2.

BEGIN DATA.

COV,ACSI1,4.00

COV,ACSI2,3.23,4.41

COV,ACSI3,2.66,2.67,3.61

COV,CUEX1,1.81,1.50,1.56,4.41

COV,CUEX2,1.85,1.62,1.63,2.63,4.84

COV,CUEX3,1.24,1.16,1.09,1.55,1.72,5.29

COV,PERQ1,3.04,2.83,2.35,2.07,1.96,1.31,3.61

COV,PERQ2,2.81,2.57,2.19,1.55,1.74,1.12,2.67,3.24

COV,PERQ3,1.73,1.64,1.32,0.89,1.05,1.41,1.62,1.62,2.89

COV,PERV1,2.66,2.49,2.12,1.40,1.43,0.95,2.22,2.01,1.25,3.24

COV,PERV2,2.99,2.86,2.42,1.52,1.50,1.01,2.34,2.14,1.31,3.05,4.84

N,,10417,10417,10417,10417,10417,10417,10417,10417,10417,10417,10417

END DATA.

SAVE OUTFILE='ACSICovData.sav'.

## SEM program Input:

Two methods are used to determine the scale of the latent variable in SEM:

1. Fix one of the path coefficients between the latent variable and the corresponding manifest variables (i.e., factor loadings) to a non-zero value (typically 1), or
2. Fix the variance of latent variables to a non-zero value (typically 1).

The first style is the AMOS default, so it is used more often. However, the second style is more convenient for estimating factor correlations. In other words, you will probably be more familiar with the following style.

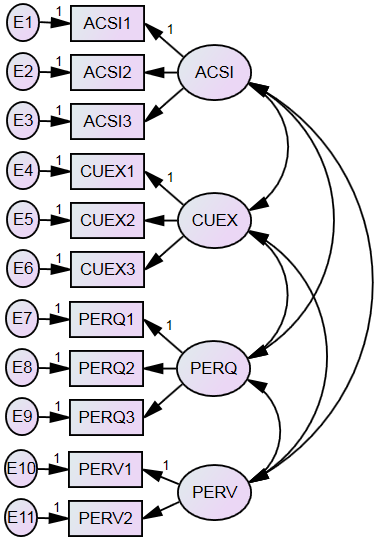


Figure 1 CFA model with default scaling in AMOS

However, we recommend the following style when assessing discriminant validity:

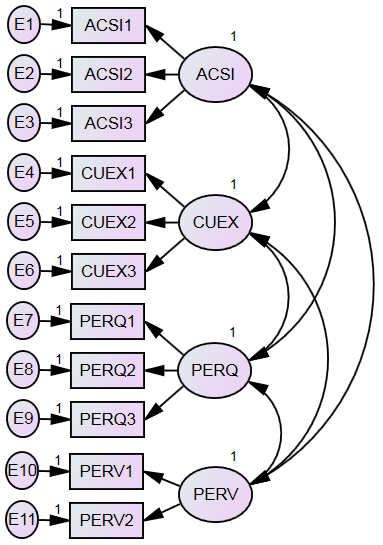


Figure 2 CFA model with alternative scaling by constraining the factor variances in AMOS

## SEM program Output:

Regression Weights: (Group number 1 - Default model)

|  |  |  | Estimate | S.E. | C.R. | P | Label |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ACSI1 | <--- | ACSI | 1.853 | .015 | 122.923 | \*\*\* |  |
| ACSI2 | <--- | ACSI | 1.745 | .017 | 103.054 | \*\*\* |  |
| ACSI3 | <--- | ACSI | 1.459 | .016 | 91.641 | \*\*\* |  |
| CUEX1 | <--- | CUEX | 1.581 | .021 | 76.575 | \*\*\* |  |
| CUEX2 | <--- | CUEX | 1.647 | .022 | 76.092 | \*\*\* |  |
| CUEX3 | <--- | CUEX | 1.038 | .024 | 42.957 | \*\*\* |  |
| PERQ1 | <--- | PERQ | 1.711 | .015 | 116.260 | \*\*\* |  |
| PERQ2 | <--- | PERQ | 1.563 | .014 | 109.737 | \*\*\* |  |
| PERQ3 | <--- | PERQ | .978 | .016 | 62.373 | \*\*\* |  |
| PERV1 | <--- | PERV | 1.643 | .014 | 115.119 | \*\*\* |  |
| PERV2 | <--- | PERV | 1.856 | .018 | 102.476 | \*\*\* |  |

Standardized Regression Weights: (Group number 1 - Default model)

|  |  |  | Estimate |
| --- | --- | --- | --- |
| ACSI1 | <--- | ACSI | .926 |
| ACSI2 | <--- | ACSI | .831 |
| ACSI3 | <--- | ACSI | .768 |
| CUEX1 | <--- | CUEX | .753 |
| CUEX2 | <--- | CUEX | .749 |
| CUEX3 | <--- | CUEX | .451 |
| PERQ1 | <--- | PERQ | .901 |
| PERQ2 | <--- | PERQ | .868 |
| PERQ3 | <--- | PERQ | .575 |
| PERV1 | <--- | PERV | .913 |
| PERV2 | <--- | PERV | .844 |

Covariances: (Group number 1 - Default model)

|  |  |  | Estimate | S.E. | C.R. | P | Label |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ACSI | <--> | CUEX | .612 | .009 | 71.842 | \*\*\* |  |
| CUEX | <--> | PERQ | .698 | .008 | 90.899 | \*\*\* |  |
| PERQ | <--> | PERV | .770 | .006 | 139.820 | \*\*\* |  |
| CUEX | <--> | PERV | .526 | .010 | 54.785 | \*\*\* |  |
| ACSI | <--> | PERQ | .957 | .002 | 384.915 | \*\*\* |  |
| ACSI | <--> | PERV | .875 | .004 | 226.975 | \*\*\* |  |

Correlations: (Group number 1 - Default model)

|  |  |  | Estimate |
| --- | --- | --- | --- |
| ACSI | <--> | CUEX | .612 |
| CUEX | <--> | PERQ | .698 |
| PERQ | <--> | PERV | .770 |
| CUEX | <--> | PERV | .526 |
| ACSI | <--> | PERQ | .957 |
| ACSI | <--> | PERV | .875 |

Variances: (Group number 1 - Default model)

|  |  |  | Estimate | S.E. | C.R. | P | Label |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ACSI |  |  | 1.000 |  |  |  |  |
| CUEX |  |  | 1.000 |  |  |  |  |
| PERQ |  |  | 1.000 |  |  |  |  |
| PERV |  |  | 1.000 |  |  |  |  |
| e1 |  |  | .566 | .013 | 43.220 | \*\*\* |  |
| e2 |  |  | 1.365 | .022 | 62.780 | \*\*\* |  |
| e3 |  |  | 1.480 | .022 | 66.229 | \*\*\* |  |
| e4 |  |  | 1.909 | .044 | 43.449 | \*\*\* |  |
| e5 |  |  | 2.128 | .048 | 44.158 | \*\*\* |  |
| e6 |  |  | 4.212 | .063 | 66.965 | \*\*\* |  |
| e7 |  |  | .682 | .015 | 46.767 | \*\*\* |  |
| e8 |  |  | .796 | .015 | 54.786 | \*\*\* |  |
| e9 |  |  | 1.933 | .028 | 69.566 | \*\*\* |  |
| e10 |  |  | .539 | .017 | 31.210 | \*\*\* |  |
| e11 |  |  | 1.396 | .028 | 50.373 | \*\*\* |  |

##### Model Fit Summary

##### CMIN

| Model | NPAR | CMIN | DF | P | CMIN/DF |
| --- | --- | --- | --- | --- | --- |
| Default model | 28 | 1614.131 | 38 | .000 | 42.477 |
| Saturated model | 66 | .000 | 0 |  |  |
| Independence model | 11 | 75019.111 | 55 | .000 | 1363.984 |

##### RMR, GFI

| Model | RMR | GFI | AGFI | PGFI |
| --- | --- | --- | --- | --- |
| Default model | .113 | .973 | .953 | .560 |
| Saturated model | .000 | 1.000 |  |  |
| Independence model | 1.837 | .272 | .127 | .227 |

##### Baseline Comparisons

| Model | NFI Delta1 | RFI rho1 | IFI Delta2 | TLI rho2 | CFI |
| --- | --- | --- | --- | --- | --- |
| Default model | .978 | .969 | .979 | .970 | .979 |
| Saturated model | 1.000 |  | 1.000 |  | 1.000 |
| Independence model | .000 | .000 | .000 | .000 | .000 |

##### RMSEA

| Model | RMSEA | LO 90 | HI 90 | PCLOSE |
| --- | --- | --- | --- | --- |
| Default model | .063 | .060 | .066 | .000 |
| Independence model | .362 | .360 | .364 | .000 |

We will next show how these results can be used to calculate all the statistics and tests explained in the article.

## Factor correlation (point estimate)

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

The advantage of fixing the variance of latent variables to 1 is that the estimated factor covariance are correlations that can be interpreted directly without any further calculation.

Table 15 Factor correlation estimates from AMOS

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .612 | 1.000 |  |  |
| PERQ | .957 | .698 | 1.000 |  |
| PERV | .875 | .526 | . 770 | 1.000 |

### Problems / Limitations

The point estimate provides only limited information about the parameter. Particularly, a single point estimate does not tell us anything about how certain we are about the estimate. A better alternative is an interval estimate in the form of a 95% confidence interval, which gives information on the maximum and minimum values of the parameter when the assumptions are met at a given confidence level.

## Factor correlation (whether the confidence interval includes 1)

### How often is it used?

AMJ 0%, JAP 0%, ORM 5.0%

### How to obtain

AMOS does not directly offer interval estimates of the factor correlation, but it provides standard error values, which can be used to easily calculate interval estimates. That is,

* 90% confidence interval (=.1) : (, )
* 95% confidence interval (=.05): (, )
* 99% confidence interval (=.01): (, )

For example, the interval estimate (=.05) of the factor correlation between ACSI and CUEX is:

.594, .630)

By repeating the above calculation, the following table can be obtained.

Table 16 Confidence intervals for correlations based on AMOS estimates

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | [.594,.630] | 1.000 |  |  |
| PERQ | [.953,.961] | [.682,.714] | 1.000 |  |
| PERV | [.867,.883] | [.506,.546] | [.758,.782] | 1.000 |

### Problems / Limitations

There is no problem with this technique itself, but the problem lies in the way we have used this technique this far. When evaluating discriminant validity, we have examined whether the interval estimates of factor correlation include one (i.e., perfect correlation). That is, if the maximum value of the confidence interval is less than 1, it is determined that there is no discriminant validity problem. This is problematic because almost all data will meet these criteria as long as the sample size is large enough. For example, in the above example, the factor correlation between ACSI-PERQ is very high, but its confidence interval does not include 1.

## Techniques using model fit indices: no comparison

### How often is it used?

AMJ 11.1%, JAP 1.4%, ORM 0%

### How to obtain

The values of the model fit indices are automatically calculated in AMOS. For example, this case the fit indices are = 1614.131, p < 0.001, GFI .973, TLI 970, CFI .979, and RMSEA .063. The value shows that the model does not fit exactly. While there are SEM model evaluation guidelines that provide cutoffs for the other indices and our values would be considered acceptable against these cutoffs, we nevertheless suggest that researchers diagnose their models to understand the source of misfit before declaring misfit acceptable (Kline, 2011, Chapter 8). However, applying this technique makes no indication of any problem in the discriminant validity of these data.

### Problems / Limitations

The fit of the proposed model has nothing to do with the discriminant validity. Assessing discriminant validity requires a well-fitting model, but the model fit itself does not inform us about discriminant validity. To assess discriminant validity using model fit indices, a comparison with other alternative models is needed. The question is which alternative model to compare.

## Techniques using model fit indices: compared to nested models with fewer factors

### How often is it used?

AMJ 29.6%, JAP 58.9%, ORM 25.0%

As far as we know, there are no guidelines-type article that recommends the use of this technique for evaluating discriminant validity. Surprisingly, however, this technique is the most commonly used technique in applied psychology.

### How to obtain

Because there is no authoritative source, this technique is being applied in a wide variety of ways. A typical method is as follows. Suppose the proposed model is composed of N factors. Then, we can construct (N - 1) -factor models, (N - 2)-factor models … and a 1-factor model by merging some of the factors into one. This technique compares the fit indices of all these (or some arbitrarily selected) alternative models with the originally proposed models.

In our ACSI example, the factor correlation between ACSI and PERQ is the highest, so you can compare an alternative three-factor model that combines the two factors into one factor with the originally proposed model. (Of course, it is also possible to review comparisons with all possible three-factor, two-factor, and one-factor models.) In other words, we construct the following model in AMOS.

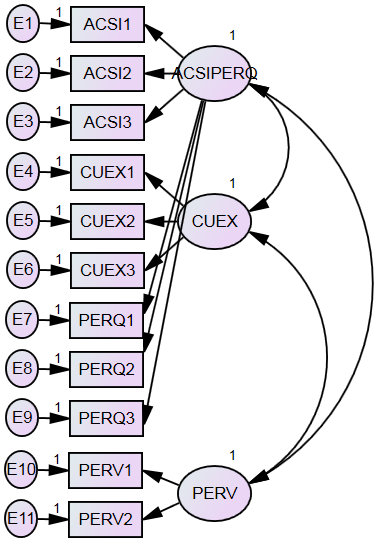


Figure 3 CFA model with two factors merged as one in AMOS

Table 17 Model fit indides for the comparison of three and four factor models

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI | RMSEA | df |  | p |
| a. Alternative 3-factor model | .961 | .948 | .083 | 41 | 2972.236 | .000 |
| b. Proposed 4-factor model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 3 | 1358.105 | .000 |

The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

### Problems / Limitations

Notice that the difference in degrees of freedom between the proposed model and the alternative model is three. Originally, our interest was a high correlation between ACSI and PERQ, but merging the two factors into one adds additional constraints. In other words, the above model is equivalent to the original model with the following three constraints.

1. The correlation between ACSI and PERQ (.957) is 1.
2. The correlation between ACSI and CUEX (.612) and the correlation between PERQ and CUEX (.698) are equal.
3. The correlation between ACSI and PERV (.875) and the correlation between PERQ and PERV (. 770) are equal.

That is, the following model:

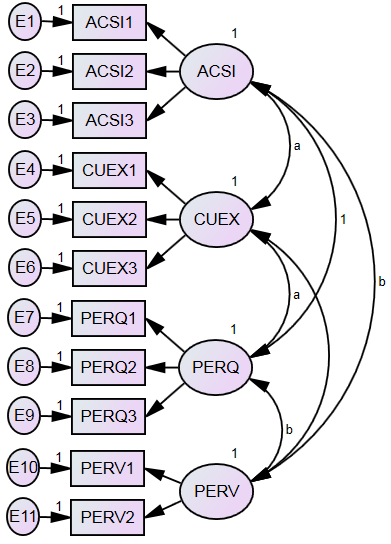


Figure 4 CFA model where two factors are constrained to be the same by adding equality constraints in AMOS

Of these, only the first constraint is truly relevant to discriminant validity, and a strategy that focuses only on the necessary constraints is needed.

## Techniques using model fit indices: comparison against model with correlation fixed at 1

### How often is it used?

AMJ 14.8%, JAP 1.4%, ORM 10.0%

### How to obtain

For all possible latent variable pairs, the model with the correlation fixed at 1 is compared with the original model. Here, we present only the model with the correlation between ACSI and PERQ is constrained to 1. That is,

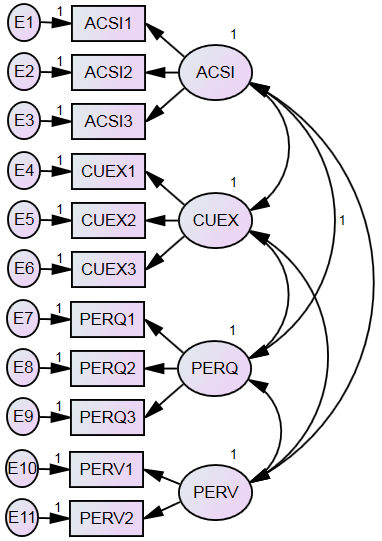


Figure 5 CFA model where two factors are constrained to be perfectly correlated in AMOS

Table 18 Model fit indices for nested model test for perfect correlation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI | RMSEA | df |  | p |
| a. A model with a fixed correlation of 1 | .974 | .963 | .069 | 39 | 1999.581 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 1 | 385.450 | .000 |

Although the correlation between ACSI and PERQ is very high at .957, the model with a fixed correlation of 1 has poorer fit indices than the original model. This test can be automated so that all correlations are compared in a single set of analyses using the “Multiple Models in a Single Analysis” functionality (Arbuckle, 2017, pp. 119–124).

### Problems / Limitations

There are no logical flaws in this technique. However, there is a practical problem that almost all data pass the criteria as long as the sample size is large enough because correlations are rarely exactly 1. Thus, applied researchers have preferred a technique that require more difficult-to-pass criteria.

## AVE: compared with the square of factor correlation

### How often is it used?

AMJ 7.4%, JAP 5.5%, ORM 5.0%

Although this technique is not used very often among organizational researchers, it is a standard technique for evaluating discriminant validity in many other business disciplines, such as marketing.

### How to obtain

The original formula of Average Variance Extracted (AVE), proposed by Fornell and Larcker (1981) is:

where is the standardized loading of indicator . Because of standardization, this can be simplified to be the mean of squared factor loadings.

For example, let's calculate ACSI’s AVE using this simpler formula:

If we use the original formula and the unstandardized coefficients, the resulting AVE will be different:

As can be seen from the above calculations, the two versions of the AVE formula usually yield close values, but are not mathematically equivalent. Instead of a mathematical proof of how the two formulas differ, we present a simple analogy. For example, assume that there are three values , , and . The original formula is to calculate , and the formula using the standardized coefficients is to calculate ++) . For the same reason, the value obtained by applying the standardized coefficients to the original formula of AVE is different from the value obtained by applying the unstandardized coefficients to the same formula. Therefore, when presenting the AVE value, we should specify whether standardized or unstandardized coefficients are used. However, note that the interpretation of AVE as variance explained is only valid if the latent variables were scaled to unit variances (which would be automatically the case in fully standardized estimates).

We will now explain how the AVE values are used in the The Fornell-Larcker criterion for assessing discriminant validity. The table below shows the AVE values using unstandardized coefficients on the diagonal (in italics), the previously reported factor correlations on the lower-triangle and the squares of the factor correlations are shown on the upper triangle (e.g.,.

Table 19 Comparison of AVEs against squared factor correlation estimates

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .375 | .916 | .766 |
| CUEX | .612 | *.444* | .487 | .277 |
| PERQ | .957 | .698 | *.632* | .593 |
| PERV | .875 | .526 | . 770 | *.773* |

The Fornell-Larcker criterion for assessing discriminant validity is that for every pair of latent variables, the square of the factor correlation must be less than the AVE values of both latent variables. The acronym AVE/SV comes from the fact that the square of the factor correlation is also called shared variance. The above example fails this criterion for three pairs of latent variables: 1) The shared variance between ACSI and PERQ is .916, which is larger than ACSI's AVE value of .713 and PERQ's AVE value of .632. 2) The shared variance between ACSI and PERV is .766, which is larger than the ACSI’s AVE value of .713. 3) The shared variance between CUEX and PERQ is .487, which is greater than the CUEX’s AVE value of .444.

### Problems / Limitations

Few methodological studies have seriously considered this technique, and the logical problems of this technique are almost unknown. Essentially, AVE is a weighted average of item-level reliabilities. While factor correlation is related to discriminant validity, as a comparison point, AVE has little relevance. Moreover, given methodological literature’s lack of attention to this technique, it is often is misapplied in a variety of ways. The most common misuse is that AVE is compared to a value other than the square of the factor correlation.

## AVE: compared with the square of scale score correlation

### How often is it used?

AMJ 3.7%, JAP 1.4%, ORM 0%

Many users are more familiar with the process of obtaining scale score correlations than factor correlations. Therefore, unlike the original proposal of Fornell and Larcker (1981a), studies comparing AVE to the square of the scale score correlation are often found.

### How to obtain

AVE values are shown on the diagonals (in italics). The previously reported scale score correlation (Table 2) is shown in the sub-diagonals. The square of the scale score correlation is displayed in the super-diagonals (e.g.,.

Table 20 Comparison of AVEs against squared scale score correlations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .235 | .667 | .585 |
| CUEX | .485 | *.444* | .303 | .163 |
| PERQ | .817 | .550 | *.632* | .416 |
| PERV | .765 | .404 | .645 | *.773* |

The Fornell-Larcker criterion was violated in only one case. The square of scale score correlation between ACSI and PERQ is .667, which is larger than PERQ's AVE value of .632.

### Problems / Limitations

It is clear that this technique is misuse, so much explanation is not necessary.

## Techniques using model fit indices: comparison against model with correlation fixed at cutoff point less than 1

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

The fact that a correlation is not exactly 1 may be a necessary condition for discriminant validity, but it is difficult to consider it a sufficient condition. In other words, a more realistic test would test against a high but not necessarily perfect correlation. This is the idea in using references values below 1. Which cutoff to use is a matter of subjective judgment, but we can consider candidates such as .85, .9, or .95. This test makes sense only if the correlation estimate is below the the cutoff; if it does not, the discriminant validity test should be considered as failed. To demonstrate this test, we compare the correlation between ACSI and PERV against a fixed cutoff of .96, which in this case is just an arbitrarily chosen cutoff that is greater than the estimated correlation of .957.

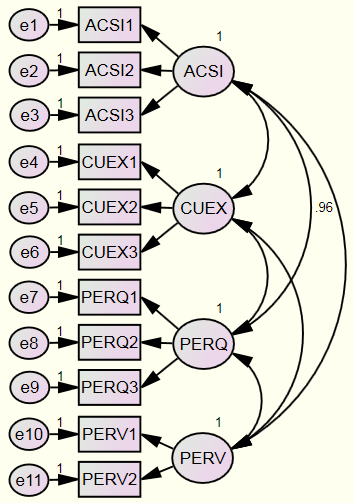


Figure 6 CFA model where two factors are constrained to be correlated at a cutoff other than 1 in AMOS

Table 21 Model fit indices for nested model test for correlation constrained to a cutoff other than 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI | RMSEA | df |  | p |
| a. A model with a fixed correlation of .96 | .979 | .970 | .062 | 39 | 1615.740 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 1 | 1.609 | .205 |

If the reference point is .96, the correlation between ACSI and PERV is not significantly less than that, so we can assess that there is a problem in the discriminant validity.

### Problems / Limitations

This technique has the advantage of being more flexible than the previous techniques, but there are also limitations. There is room for criticism that the user must determine the reference value, which is an arbitrary choice. While AMOS provides a feature to do several model comparisons in one analysis, specifying the constraints one at a time is tedious.

## Structure coefficients (obtained from CFA)

### How often is it used?

AMJ 7.4%, JAP 0%, ORM 0%

### How to obtain

A typical CFA is an independent clusters (IC) model, where each item loads on just one factor and the loading value with other factors is zero. In the case of a CFA model specification, the term loading invariably refers to the factor pattern coefficients because the factor structure coefficients are not model parameters that are estimated, but are something that can be calculated post-estimation.

In AMOS, structure coefficients can be obtained by choosing *Standardized estimates* and *All implied moments* in the output dialog. However, these can also be calculated by hand. Factor structure coefficients are the matrix multiplication of factor pattern coefficients and factor correlations. That is, the (i, j) th element of the third matrix is obtained by multiplying the (i, k) element of the first matrix by the element (k, j) of the second matrix, and summing it over all k. For example, .567 in factor structure coefficients is obtained by the following calculation. **.567** = **.926** \* **.612**+ **.000** \* **1.000** + **.000** \* **.698** + **.000** \* **.526**. One convenient way to calculate this matrix product is to use Microsoft Excel’s MMULT worksheet function.

We will now demonstrate the calculation. Let's determine the cross-loading according to some rules discussed above. First, absolute comparison. Let's choose .4 as the cutoff point. There are cross-loadings in all items except CUEX3. Second, row comparison. There is no cross-loading on any item. Third, column comparison. Cross-loadings can be found in five items: ACSI2, ACSI3, CUEX3, PERQ2, and PERQ3.

Table 22 Calculating factor structure coefficients from AMOS output

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A.** Factor pattern coefficients | | | | **B.** Factor correlations | | | | **C.** Factor structure coefficients | | | |
| ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV |
| **.926** | **.000** | **.000** | **.000** | 1.000 | **.612** | .957 | .875 | 0.926 | **0.567** | 0.886 | 0.810 |
| .831 | .000 | .000 | .000 | .612 | **1.000** | .698 | .526 | 0.831 | 0.509 | 0.795 | 0.727 |
| .768 | .000 | .000 | .000 | .957 | **.698** | 1.000 | .770 | 0.768 | 0.470 | 0.735 | 0.672 |
| .000 | .753 | .000 | .000 | .875 | **.526** | . 770 | 1.000 | 0.461 | 0.753 | 0.526 | 0.396 |
| .000 | .749 | .000 | .000 |  |  |  |  | 0.458 | 0.749 | 0.523 | 0.394 |
| .000 | .451 | .000 | .000 |  |  |  |  | 0.276 | 0.451 | 0.315 | 0.237 |
| .000 | .000 | .901 | .000 |  |  |  |  | 0.862 | 0.629 | 0.901 | 0.694 |
| .000 | .000 | .868 | .000 |  |  |  |  | 0.831 | 0.606 | 0.868 | 0.668 |
| .000 | .000 | .575 | .000 |  |  |  |  | 0.550 | 0.401 | 0.575 | 0.443 |
| .000 | .000 | .000 | .913 |  |  |  |  | 0.799 | 0.480 | 0.703 | 0.913 |
| .000 | .000 | .000 | .844 |  |  |  |  | 0.739 | 0.444 | 0.650 | 0.844 |

### Problems / Limitations

As discussed earlier, checking for cross-loadings is not a technique that closely matches the definition of discriminant validity.

## CICFA(.9): the proposed technique

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

There are no studies that formally proposed this technique.

### How to obtain

The method of obtaining the upper limit of factor correlation in AMOS has been described above (i.e., Table 16). The difference between the existing technique and CICFA (.9) is what this upper limit is compared to. The existing technique evaluates that there is a problem with discriminant validity if the upper limit is larger than 1. Our concern is not whether the correlation is exactly 1, but whether it is a sufficiently low number of 1, so the existing technique does not fit well with the definition of discriminant validity. We suggest using a number less than 1 as a cutoff. CICFA (.9) evaluates that there is a problem with discriminant validity if the upper limit is larger than .9. The upper limit of the correlation between ACSI-PERQ is .961, which does not pass this criterion.

### Problems / Limitations

The choice of a cutoff is inevitable for a dichotomous (i.e., yes or no) decision, but there is room for criticism that any cutoff is arbitrary. More research is needed as to which cutoff is best, and different cutoffs may be considered depending on the purpose of the study (e.g., CICFA(.85)).

# Techniques that require SEM software: LISREL version

## SEM program Input:

Two methods are used to determine the scale of the latent variable in SEM:

1. Fix one of the path coefficients between the latent variable and the corresponding manifest variables (i.e., factor loadings) to a non-zero value (typically 1), or
2. Fix the variance of latent variables to a non-zero value (typically 1).

Unlike other SEM software, LISREL does not have a default option to determine the scale. Most users choose the first style, which is also the default option in most SEM software. However, the second style is more convenient for estimating factor correlations. In other words, you will probably be more familiar with the following style.

VA 1 LX 1 1 LX 4 2 LX 7 3 LX 10 4

FR LX 2 1 LX 3 1 LX 5 2 LX 6 2 LX 8 3 LX 9 3 LX 11 4

However, we recommend the following style when assessing discriminant validity:

VA 1 PH 1 1 PH 2 2 PH 3 3 PH 4 4

FR LX 1 1 LX 2 1 LX 3 1 LX 4 2 LX 5 2 LX 6 2 LX 7 3 LX 8 3 LX 9 3 LX 10 4 LX 11 4

The complete input is below.

DA NI=11 NO=10417 MA=CM

CM SY

4.00

3.23 4.41

2.66 2.67 3.61

1.81 1.50 1.56 4.41

1.85 1.62 1.63 2.63 4.84

1.24 1.16 1.09 1.55 1.72 5.29

3.04 2.83 2.35 2.07 1.96 1.31 3.61

2.81 2.57 2.19 1.55 1.74 1.12 2.67 3.24

1.73 1.64 1.32 0.89 1.05 1.41 1.62 1.62 2.89

2.66 2.49 2.12 1.40 1.43 0.95 2.22 2.01 1.25 3.24

2.99 2.86 2.42 1.52 1.50 1.01 2.34 2.14 1.31 3.05 4.84

MO NX=11 NK=4 PH=FR

VA 1 PH 1 1 PH 2 2 PH 3 3 PH 4 4

FR LX 1 1 LX 2 1 LX 3 1 LX 4 2 LX 5 2 LX 6 2 LX 7 3 LX 8 3 LX 9 3 LX 10 4 LX 11 4

OU SC ME=ML ND=3

## SEM program Output:

Number of Iterations = 5

LISREL Estimates (Maximum Likelihood)

LAMBDA-X

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

VAR 1 1.853 - - - - - -

(0.015)

122.923

VAR 2 1.745 - - - - - -

(0.017)

103.054

VAR 3 1.459 - - - - - -

(0.016)

91.641

VAR 4 - - 1.581 - - - -

(0.021)

76.575

VAR 5 - - 1.647 - - - -

(0.022)

76.092

VAR 6 - - 1.038 - - - -

(0.024)

42.957

VAR 7 - - - - 1.711 - -

(0.015)

116.260

VAR 8 - - - - 1.563 - -

(0.014)

109.737

VAR 9 - - - - 0.978 - -

(0.016)

62.373

VAR 10 - - - - - - 1.644

(0.014)

115.119

VAR 11 - - - - - - 1.856

(0.018)

102.476

PHI

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

KSI 1 1.000

KSI 2 0.612 1.000

(0.009)

71.842

KSI 3 0.957 0.698 1.000

(0.002) (0.008)

384.915 90.899

KSI 4 0.875 0.526 0.770 1.000

(0.004) (0.010) (0.006)

226.975 54.785 139.820

THETA-DELTA

VAR 1 VAR 2 VAR 3 VAR 4 VAR 5 VAR 6

-------- -------- -------- -------- -------- --------

0.567 1.365 1.480 1.909 2.128 4.213

(0.013) (0.022) (0.022) (0.044) (0.048) (0.063)

43.220 62.780 66.229 43.449 44.158 66.965

THETA-DELTA

VAR 7 VAR 8 VAR 9 VAR 10 VAR 11

-------- -------- -------- -------- --------

0.682 0.796 1.934 0.539 1.396

(0.015) (0.015) (0.028) (0.017) (0.028)

46.767 54.786 69.566 31.210 50.373

Squared Multiple Correlations for X - Variables

VAR 1 VAR 2 VAR 3 VAR 4 VAR 5 VAR 6

-------- -------- -------- -------- -------- --------

0.858 0.690 0.590 0.567 0.560 0.204

Squared Multiple Correlations for X - Variables

VAR 7 VAR 8 VAR 9 VAR 10 VAR 11

-------- -------- -------- -------- --------

0.811 0.754 0.331 0.834 0.712

Goodness of Fit Statistics

Degrees of Freedom = 38

Minimum Fit Function Chi-Square = 1614.131 (P = 0.0)

Normal Theory Weighted Least Squares Chi-Square = 1586.835 (P = 0.0)

Estimated Non-centrality Parameter (NCP) = 1548.835

90 Percent Confidence Interval for NCP = (1422.217 ; 1682.823)

Minimum Fit Function Value = 0.155

Population Discrepancy Function Value (F0) = 0.149

90 Percent Confidence Interval for F0 = (0.137 ; 0.162)

Root Mean Square Error of Approximation (RMSEA) = 0.0626

90 Percent Confidence Interval for RMSEA = (0.0599 ; 0.0652)

P-Value for Test of Close Fit (RMSEA < 0.05) = 0.378

Expected Cross-Validation Index (ECVI) = 0.158

90 Percent Confidence Interval for ECVI = (0.146 ; 0.171)

ECVI for Saturated Model = 0.0127

ECVI for Independence Model = 7.204

Chi-Square for Independence Model with 55 Degrees of Freedom = 75019.111

Independence AIC = 75041.111

Model AIC = 1642.835

Saturated AIC = 132.000

Independence CAIC = 75131.874

Model CAIC = 1873.869

Saturated CAIC = 676.579

Root Mean Square Residual (RMR) = 0.113

Standardized RMR = 0.0285

Goodness of Fit Index (GFI) = 0.973

Adjusted Goodness of Fit Index (AGFI) = 0.953

Parsimony Goodness of Fit Index (PGFI) = 0.560

Normed Fit Index (NFI) = 0.978

Non-Normed Fit Index (NNFI) = 0.970

Parsimony Normed Fit Index (PNFI) = 0.676

Comparative Fit Index (CFI) = 0.979

Incremental Fit Index (IFI) = 0.979

Relative Fit Index (RFI) = 0.969

Critical N (CN) = 395.684

Standardized Solution

LAMBDA-X

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

VAR 1 1.853 - - - - - -

VAR 2 1.745 - - - - - -

VAR 3 1.459 - - - - - -

VAR 4 - - 1.581 - - - -

VAR 5 - - 1.647 - - - -

VAR 6 - - 1.038 - - - -

VAR 7 - - - - 1.711 - -

VAR 8 - - - - 1.563 - -

VAR 9 - - - - 0.978 - -

VAR 10 - - - - - - 1.644

VAR 11 - - - - - - 1.856

PHI

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

KSI 1 1.000

KSI 2 0.612 1.000

KSI 3 0.957 0.698 1.000

KSI 4 0.875 0.526 0.770 1.000

Completely Standardized Solution

LAMBDA-X

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

VAR 1 0.926 - - - - - -

VAR 2 0.831 - - - - - -

VAR 3 0.768 - - - - - -

VAR 4 - - 0.753 - - - -

VAR 5 - - 0.749 - - - -

VAR 6 - - 0.451 - - - -

VAR 7 - - - - 0.901 - -

VAR 8 - - - - 0.868 - -

VAR 9 - - - - 0.575 - -

VAR 10 - - - - - - 0.913

VAR 11 - - - - - - 0.844

PHI

KSI 1 KSI 2 KSI 3 KSI 4

-------- -------- -------- --------

KSI 1 1.000

KSI 2 0.612 1.000

KSI 3 0.957 0.698 1.000

KSI 4 0.875 0.526 0.770 1.000

THETA-DELTA

VAR 1 VAR 2 VAR 3 VAR 4 VAR 5 VAR 6

-------- -------- -------- -------- -------- --------

0.142 0.310 0.410 0.433 0.440 0.796

THETA-DELTA

VAR 7 VAR 8 VAR 9 VAR 10 VAR 11

-------- -------- -------- -------- --------

0.189 0.246 0.669 0.166 0.288

We will next show how these results can be used to calculate all the statistics and tests explained in the article.

## Factor correlation (point estimate)

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

The advantage of fixing the variance of latent variables to 1 is that the estimated factor covariance are correlations that can be interpreted directly without any further calculation.

Table 23 Factor correlation estimates from LISREL

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .612 | 1.000 |  |  |
| PERQ | .957 | .698 | 1.000 |  |
| PERV | .875 | .526 | . 770 | 1.000 |

### Problems / Limitations

The point estimate provides only limited information about the parameter. Particularly, a single point estimate does not tell us anything about how certain we are about the estimate. A better alternative is an interval estimate in the form of a 95% confidence interval, which gives information on the maximum and minimum values of the parameter when the assumptions are met at a given confidence level.

## Factor correlation (whether the confidence interval includes 1)

### How often is it used?

AMJ 0%, JAP 0%, ORM 5.0%

### How to obtain

LISREL does not directly offer interval estimates of the factor correlation, but it provides standard error values, which can be used to easily calculate interval estimates. That is,

* 90% confidence interval (=.1) : (, )
* 95% confidence interval (=.05): (, )
* 99% confidence interval (=.01): (, )

For example, the interval estimate (=.05) of the factor correlation between ACSI and CUEX is:

.594, .630)

By repeating the above calculation, the following table can be obtained.

Table 24 Confidence intervals for correlations based on LISREL estimates

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | [.594,.630] | 1.000 |  |  |
| PERQ | [.953,.961] | [.682,.714] | 1.000 |  |
| PERV | [.867,.883] | [.506,.546] | [.758,.782] | 1.000 |

### Problems / Limitations

There is no problem with this technique itself, but the problem lies in the way we have used this technique this far. When evaluating discriminant validity, we have examined whether the interval estimates of factor correlation include one (i.e., perfect correlation). That is, if the maximum value of the confidence interval is less than 1, it is determined that there is no discriminant validity problem. This is problematic because almost all data will meet these criteria as long as the sample size is large enough. For example, in the above example, the factor correlation between ACSI-PERQ is very high, but its confidence interval does not include 1.

## Techniques using model fit indices: no comparison

### How often is it used?

AMJ 11.1%, JAP 1.4%, ORM 0%

### How to obtain

The values of the model fit indices are automatically calculated in LISREL. For example, this case the fit indices are = 1614.131, p = 0.0, GFI .973, TLI (referred to as NNFI in LISREL) 970, CFI .979, and RMSEA .0626. The value shows that the model does not fit exactly. While there are SEM model evaluation guidelines that provide cutoffs for the other indices and our values would be considered acceptable against these cutoffs, we nevertheless suggest that researchers diagnose their models to understand the source of misfit before declaring misfit acceptable (Kline, 2011, Chapter 8). However, applying this technique makes no indication of any problem in the discriminant validity of these data.

### Problems / Limitations

The fit of the proposed model has nothing to do with the discriminant validity. Assessing discriminant validity requires a well-fitting model, but the model fit itself does not inform us about discriminant validity. To assess discriminant validity using model fit indices, a comparison with other alternative models is needed. The question is which alternative model to compare.

## Techniques using model fit indices: compared to nested models with fewer factors

### How often is it used?

AMJ 29.6%, JAP 58.9%, ORM 25.0%

As far as we know, there are no guidelines-type article that recommends the use of this technique for evaluating discriminant validity. Surprisingly, however, this technique is the most commonly used technique in applied psychology.

### How to obtain

Because there is no authoritative source, this technique is being applied in a wide variety of ways. A typical method is as follows. Suppose the proposed model is composed of N factors. Then, we can construct (N - 1) -factor models, (N - 2)-factor models … and a 1-factor model by merging some of the factors into one. This technique compares the fit indices of all these (or some arbitrarily selected) alternative models with the originally proposed models.

In our ACSI example, the factor correlation between ACSI and PERQ is the highest, so you can compare an alternative three-factor model that combines the two factors into one factor with the originally proposed model. (Of course, it is also possible to review comparisons with all possible three-factor, two-factor, and one-factor models.) In other words, we construct the following model in LISREL.

MO NX=11 NK=3 PH=FR

VA 1 PH 1 1 PH 2 2 PH 3 3

FR LX 1 1 LX 2 1 LX 3 1 LX 4 2 LX 5 2 LX 6 2 LX 7 1 LX 8 1 LX 9 1 LX 10 3 LX 11 3

Table 25 Model fit indides for the comparison of three and four factor models

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. Alternative 3-factor model | .961 | .948 | .083 | 41 | 2972.236 | .000 |
| b. Proposed 4-factor model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 3 | 1358.105 | .000 |

The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

### Problems / Limitations

Notice that the difference in degrees of freedom between the proposed model and the alternative model is three. Originally, our interest was a high correlation between ACSI and PERQ, but merging the two factors into one adds additional constraints. In other words, the above model is equivalent to the original model with the following three constraints.

1. The correlation between ACSI and PERQ (.957) is 1.
2. The correlation between ACSI and CUEX (.612) and the correlation between PERQ and CUEX (.698) are equal.
3. The correlation between ACSI and PERV (.875) and the correlation between PERQ and PERV (. 770) are equal.

That is, add the following lines to the original input.

CO PH 1 3 = 1

EQ PH 1 2 PH 2 3

EQ PH 1 4 PH 3 4

Of these, only the first constraint is truly relevant to discriminant validity, and a strategy that focuses only on the necessary constraints is needed.

## Techniques using model fit indices: comparison against model with correlation fixed at 1

### How often is it used?

AMJ 14.8%, JAP 1.4%, ORM 10.0%

### How to obtain

For all possible latent variable pairs, the model with the correlation fixed at 1 is compared with the original model. Here, we present only the model with the correlation between ACSI and PERQ is constrained to 1. That is, add the following line to the original input.

CO PH 1 3 = 1

Table 26 Model fit indices for nested model test for perfect correlation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. A model with a fixed correlation of 1 | .974 | .963 | .069 | 39 | 1999.581 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 1 | 385.450 | .000 |

Although the correlation between ACSI and PERQ is very high at .957, the model with a fixed correlation of 1 has poorer fit indices than the original model.

### Problems / Limitations

There are no logical flaws in this technique. However, there is a practical problem that almost all data pass the criteria as long as the sample size is large enough because correlations are rarely exactly 1. Thus, applied researchers have preferred a technique that require more difficult-to-pass criteria.

## AVE: compared with the square of factor correlation

### How often is it used?

AMJ 7.4%, JAP 5.5%, ORM 5.0%

Although this technique is not used very often among organizational researchers, it is a standard technique for evaluating discriminant validity in many other business disciplines, such as marketing.

### How to obtain

The original formula of Average Variance Extracted (AVE), proposed by Fornell and Larcker (1981) is:

where is the standardized loading of indicator . Because of standardization, this can be simplified to be the mean of squared factor loadings.

For example, let's calculate ACSI’s AVE using this simpler formula:

If we use the original formula and the unstandardized coefficients, the resulting AVE will be different:

As can be seen from the above calculations, the two versions of the AVE formula usually yield close values, but are not mathematically equivalent. Instead of a mathematical proof of how the two formulas differ, we present a simple analogy. For example, assume that there are three values , , and . The original formula is to calculate , and the formula using the standardized coefficients is to calculate ++) . For the same reason, the value obtained by applying the standardized coefficients to the original formula of AVE is different from the value obtained by applying the unstandardized coefficients to the same formula. Therefore, when presenting the AVE value, we should specify whether standardized or unstandardized coefficients are used. However, note that the interpretation of AVE as variance explained is only valid if the latent variables were scaled to unit variances (which would be automatically the case in fully standardized estimates).

We will now explain how the AVE values are used in the The Fornell-Larcker criterion for assessing discriminant validity. The table below shows the AVE values using unstandardized coefficients on the diagonal (in italics), the previously reported factor correlations on the lower-triangle and the squares of the factor correlations are shown on the upper triangle (e.g.,.

Table 27 Comparison of AVEs against squared factor correlation estimates

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .375 | .916 | .766 |
| CUEX | .612 | *.444* | .487 | .277 |
| PERQ | .957 | .698 | *.632* | .593 |
| PERV | .875 | .526 | . 770 | *.773* |

The Fornell-Larcker criterion for assessing discriminant validity is that for every pair of latent variables, the square of the factor correlation must be less than the AVE values of both latent variables. The acronym AVE/SV comes from the fact that the square of the factor correlation is also called shared variance. The above example fails this criterion for three pairs of latent variables: 1) The shared variance between ACSI and PERQ is .916, which is larger than ACSI's AVE value of .713 and PERQ's AVE value of .632. 2) The shared variance between ACSI and PERV is .766, which is larger than the ACSI’s AVE value of .713. 3) The shared variance between CUEX and PERQ is .487, which is greater than the CUEX’s AVE value of .444.

### Problems / Limitations

Few methodological studies have seriously considered this technique, and the logical problems of this technique are almost unknown. Essentially, AVE is a weighted average of item-level reliabilities. While factor correlation is related to discriminant validity, as a comparison point, AVE has little relevance. Moreover, given methodological literature’s lack of attention to this technique, it is often is misapplied in a variety of ways. The most common misuse is that AVE is compared to a value other than the square of the factor correlation.

## AVE: compared with the square of scale score correlation

### How often is it used?

AMJ 3.7%, JAP 1.4%, ORM 0%

Many users are more familiar with the process of obtaining scale score correlations than factor correlations. Therefore, unlike the original proposal of Fornell and Larcker (1981a), studies comparing AVE to the square of the scale score correlation are often found.

### How to obtain

AVE values are shown on the diagonals (in italics). The previously reported scale score correlation (Table 2) is shown in the sub-diagonals. The square of the scale score correlation is displayed in the super-diagonals (e.g.,.

Table 28 Comparison of AVEs against squared scale score correlations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .235 | .667 | .585 |
| CUEX | .485 | *.444* | .303 | .163 |
| PERQ | .817 | .550 | *.632* | .416 |
| PERV | .765 | .404 | .645 | *.773* |

The Fornell-Larcker criterion was violated in only one case. The square of scale score correlation between ACSI and PERQ is .667, which is larger than PERQ's AVE value of .632.

### Problems / Limitations

It is clear that this technique is misuse, so much explanation is not necessary.

## Techniques using model fit indices: comparison against model with correlation fixed at cutoff point less than 1

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

The fact that a correlation is not exactly 1 may be a necessary condition for discriminant validity, but it is difficult to consider it a sufficient condition. In other words, a more realistic test would test against a high but not necessarily perfect correlation. This is the idea in using references values below 1. Which cutoff to use is a matter of subjective judgment, but we can consider candidates such as .85, .9, or .95. This test makes sense only if the correlation estimate is below the the cutoff; if it does not, the discriminant validity test should be considered as failed. To demonstrate this test, we compare the correlation between ACSI and PERV against a fixed cutoff of .96, which in this case is just an arbitrarily chosen cutoff that is greater than the estimated correlation of .957. That is, add the following line to the original input.

CO PH 1 3 = .96

Table 29 Model fit indices for nested model test for correlation constrained to a cutoff other than 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. A model with a fixed correlation of .96 | .979 | .970 | .062 | 39 | 1615.740 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.131 | .000 |
| Difference (a - b) |  |  |  | 1 | 1.609 | .205 |

If the reference point is .96, the correlation between ACSI and PERV is not significantly less than that, so we can assess that there is a problem in the discriminant validity.

### Problems / Limitations

This technique has the advantage of being more flexible than the previous techniques, but there are also limitations. There is room for criticism that the user must determine the reference value, which is an arbitrary choice. LISREL does not provide a feature to do several model comparisons in one analysis, and specifying the constraints one at a time is tedious.

## Structure coefficients (obtained from CFA)

### How often is it used?

AMJ 7.4%, JAP 0%, ORM 0%

### How to obtain

A typical CFA is an independent clusters (IC) model, where each item loads on just one factor and the loading value with other factors is zero. In the case of a CFA model specification, the term loading invariably refers to the factor pattern coefficients because the factor structure coefficients are not model parameters that are estimated, but are something that can be calculated post-estimation.

While LISREL does not provide structure coefficients directly, we explain how to obtain them. Factor structure coefficients are the matrix multiplication of factor pattern coefficients and factor correlations. That is, the (i, j) th element of the third matrix is obtained by multiplying the (i, k) element of the first matrix by the element (k, j) of the second matrix, and summing it over all k. For example, .567 in factor structure coefficients is obtained by the following calculation. **.567** = **.926** \* **.612**+ **.000** \* **1.000** + **.000** \* **.698** + **.000** \* **.526**. One convenient way to calculate this matrix product is to use Microsoft Excel’s MMULT worksheet function.

We will now demonstrate the calculation. Let's determine the cross-loading according to some rules discussed above. First, absolute comparison. Let's choose .4 as the cutoff point. There are cross-loadings in all items except CUEX3. Second, row comparison. There is no cross-loading on any item. Third, column comparison. Cross-loadings can be found in five items: ACSI2, ACSI3, CUEX3, PERQ2, and PERQ3.

Table 30 Calculating factor structure coefficients from LISREL output

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A.** Factor pattern coefficients | | | | **B.** Factor correlations | | | | **C.** Factor structure coefficients | | | |
| ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV |
| **.926** | **.000** | **.000** | **.000** | 1.000 | **.612** | .957 | .875 | 0.926 | **0.567** | 0.886 | 0.810 |
| .831 | .000 | .000 | .000 | .612 | **1.000** | .698 | .526 | 0.831 | 0.509 | 0.795 | 0.727 |
| .768 | .000 | .000 | .000 | .957 | **.698** | 1.000 | .770 | 0.768 | 0.470 | 0.735 | 0.672 |
| .000 | .753 | .000 | .000 | .875 | **.526** | . 770 | 1.000 | 0.461 | 0.753 | 0.526 | 0.396 |
| .000 | .749 | .000 | .000 |  |  |  |  | 0.458 | 0.749 | 0.523 | 0.394 |
| .000 | .451 | .000 | .000 |  |  |  |  | 0.276 | 0.451 | 0.315 | 0.237 |
| .000 | .000 | .901 | .000 |  |  |  |  | 0.862 | 0.629 | 0.901 | 0.694 |
| .000 | .000 | .868 | .000 |  |  |  |  | 0.831 | 0.606 | 0.868 | 0.668 |
| .000 | .000 | .575 | .000 |  |  |  |  | 0.550 | 0.401 | 0.575 | 0.443 |
| .000 | .000 | .000 | .913 |  |  |  |  | 0.799 | 0.480 | 0.703 | 0.913 |
| .000 | .000 | .000 | .844 |  |  |  |  | 0.739 | 0.444 | 0.650 | 0.844 |

### Problems / Limitations

As discussed earlier, checking for cross-loadings is not a technique that closely matches the definition of discriminant validity.

## CICFA(.9): the proposed technique

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

There are no studies that formally proposed this technique.

### How to obtain

The method of obtaining the upper limit of factor correlation in LISREL has been described above (i.e.,Table 24). The difference between the existing technique and CICFA (.9) is what this upper limit is compared to. The existing technique evaluates that there is a problem with discriminant validity if the upper limit is larger than 1. Our concern is not whether the correlation is exactly 1, but whether it is a sufficiently low number of 1, so the existing technique does not fit well with the definition of discriminant validity. We suggest using a number less than 1 as a cutoff. CICFA (.9) evaluates that there is a problem with discriminant validity if the upper limit is larger than .9. The upper limit of the correlation between ACSI-PERQ is .961, which does not pass this criterion.

### Problems / Limitations

The choice of a cutoff is inevitable for a dichotomous (i.e., yes or no) decision, but there is room for criticism that any cutoff is arbitrary. More research is needed as to which cutoff is best, and different cutoffs may be considered depending on the purpose of the study (e.g., CICFA(.85)).

# Techniques that require SEM software: Mplus version

Input data are provided as ACSICovData.dat and the Mplus file is ACSITutorial.inp in <https://github.com/eunscho/DiscriminantValidityTutorial>. The input data can also be created with a text editor (e.g. Notepad):

4.00

3.23 4.41

2.66 2.67 3.61

1.81 1.50 1.56 4.41

1.85 1.62 1.63 2.63 4.84

1.24 1.16 1.09 1.55 1.72 5.29

3.04 2.83 2.35 2.07 1.96 1.31 3.61

2.81 2.57 2.19 1.55 1.74 1.12 2.67 3.24

1.73 1.64 1.32 0.89 1.05 1.41 1.62 1.62 2.89

2.66 2.49 2.12 1.40 1.43 0.95 2.22 2.01 1.25 3.24

2.99 2.86 2.42 1.52 1.50 1.01 2.34 2.14 1.31 3.05 4.84

## SEM program Input:

Two methods are used to determine the scale of the latent variable in SEM:

1. Fix one of the path coefficients between the latent variable and the corresponding manifest variables (i.e., factor loadings) to a non-zero value (typically 1), or
2. Fix the variance of latent variables to a non-zero value (typically 1).

The first style is the Mplus default, so it is used more often. However, the second style is more convenient for estimating factor correlation. In other words, you will probably be more familiar with the following style.

MODEL:

! Mplus automatically fixes the first loadings to 1

ACSI BY ACSI1 ACSI2 ACSI3;

CUEX BY CUEX1 CUEX2 CUEX3;

PERQ BY PERQ1 PERQ2 PERQ3;

PERV BY PERV1 PERV2;

However, we recommend the following style.

MODEL:

! Free the first loadings with stars (\*)

ACSI BY ACSI1\* ACSI2 ACSI3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERQ BY PERQ1\* PERQ2 PERQ3;

PERV BY PERV1\* PERV2;

! Constrain the factor variances

ACSI@1;

CUEX@1;

PERQ@1;

PERV@1;

The complete input is below.

TITLE: Unconstrained CFA with confidence intervals

DATA:

FILE IS ACSICovData.dat;

TYPE = COVARIANCE;

NOBSERVATIONS = 10417;

VARIABLE:

NAMES ARE ACSI1 ACSI2 ACSI3 CUEX1 CUEX2 CUEX3 PERQ1 PERQ2 PERQ3 PERV1 PERV2;

MODEL:

! Free the first loadings with stars (\*)

ACSI BY ACSI1\* ACSI2 ACSI3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERQ BY PERQ1\* PERQ2 PERQ3;

PERV BY PERV1\* PERV2;

! Constrain the factor variances

ACSI@1;

CUEX@1;

PERQ@1;

PERV@1;

OUTPUT:

! Output the confidence intervals

CINTERVAL;

! Show standardized coefficients to calculate AVE value

STAND;

## SEM program Output:

MODEL FIT INFORMATION

Number of Free Parameters 28

Loglikelihood

H0 Value -204807.453

H1 Value -204000.310

Information Criteria

Akaike (AIC) 409670.906

Bayesian (BIC) 409873.940

Sample-Size Adjusted BIC 409784.960

(n\* = (n + 2) / 24)

Chi-Square Test of Model Fit

Value 1614.286

Degrees of Freedom 38

P-Value 0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate 0.063

90 Percent C.I. 0.060 0.066

Probability RMSEA <= .05 0.000

CFI/TLI

CFI 0.979

TLI 0.970

Chi-Square Test of Model Fit for the Baseline Model

Value 75026.313

Degrees of Freedom 55

P-Value 0.0000

SRMR (Standardized Root Mean Square Residual)

Value 0.029

MODEL RESULTS

Two-Tailed

Estimate S.E. Est./S.E. P-Value

ACSI BY

ACSI1 1.853 0.015 122.929 0.000

ACSI2 1.745 0.017 103.059 0.000

ACSI3 1.459 0.016 91.645 0.000

CUEX BY

CUEX1 1.581 0.021 76.578 0.000

CUEX2 1.647 0.022 76.095 0.000

CUEX3 1.038 0.024 42.958 0.000

PERQ BY

PERQ1 1.711 0.015 116.266 0.000

PERQ2 1.563 0.014 109.742 0.000

PERQ3 0.978 0.016 62.376 0.000

PERV BY

PERV1 1.643 0.014 115.125 0.000

PERV2 1.856 0.018 102.481 0.000

CUEX WITH

ACSI 0.612 0.009 71.846 0.000

PERQ WITH

ACSI 0.957 0.002 384.932 0.000

CUEX 0.698 0.008 90.903 0.000

PERV WITH

ACSI 0.875 0.004 226.986 0.000

CUEX 0.526 0.010 54.787 0.000

PERQ 0.770 0.006 139.827 0.000

Variances

ACSI 1.000 0.000 999.000 999.000

CUEX 1.000 0.000 999.000 999.000

PERQ 1.000 0.000 999.000 999.000

PERV 1.000 0.000 999.000 999.000

Residual Variances

ACSI1 0.566 0.013 43.222 0.000

ACSI2 1.365 0.022 62.783 0.000

ACSI3 1.480 0.022 66.232 0.000

CUEX1 1.909 0.044 43.451 0.000

CUEX2 2.128 0.048 44.160 0.000

CUEX3 4.212 0.063 66.969 0.000

PERQ1 0.682 0.015 46.769 0.000

PERQ2 0.796 0.015 54.789 0.000

PERQ3 1.933 0.028 69.570 0.000

PERV1 0.539 0.017 31.211 0.000

PERV2 1.396 0.028 50.375 0.000

STANDARDIZED MODEL RESULTS

STDYX Standardization

Two-Tailed

Estimate S.E. Est./S.E. P-Value

ACSI BY

ACSI1 0.926 0.002 467.258 0.000

ACSI2 0.831 0.003 246.695 0.000

ACSI3 0.768 0.004 177.602 0.000

CUEX BY

CUEX1 0.753 0.007 111.138 0.000

CUEX2 0.749 0.007 110.101 0.000

CUEX3 0.451 0.009 48.945 0.000

PERQ BY

PERQ1 0.901 0.003 357.328 0.000

PERQ2 0.868 0.003 295.676 0.000

PERQ3 0.575 0.007 83.106 0.000

PERV BY

PERV1 0.913 0.003 295.051 0.000

PERV2 0.844 0.004 226.294 0.000

CUEX WITH

ACSI 0.612 0.009 71.846 0.000

PERQ WITH

ACSI 0.957 0.002 384.932 0.000

CUEX 0.698 0.008 90.903 0.000

PERV WITH

ACSI 0.875 0.004 226.986 0.000

CUEX 0.526 0.010 54.787 0.000

PERQ 0.770 0.006 139.827 0.000

CONFIDENCE INTERVALS OF MODEL RESULTS

Lower .5% Lower 2.5% Lower 5% Estimate Upper 5% Upper 2.5% Upper .5%

ACSI BY

ACSI1 1.814 1.823 1.828 1.853 1.878 1.882 1.892

ACSI2 1.701 1.712 1.717 1.745 1.773 1.778 1.788

ACSI3 1.418 1.428 1.433 1.459 1.485 1.491 1.500

CUEX BY

CUEX1 1.528 1.541 1.547 1.581 1.615 1.622 1.634

CUEX2 1.591 1.604 1.611 1.647 1.682 1.689 1.702

CUEX3 0.976 0.990 0.998 1.038 1.078 1.085 1.100

PERQ BY

PERQ1 1.673 1.682 1.687 1.711 1.735 1.740 1.749

PERQ2 1.526 1.535 1.540 1.563 1.587 1.591 1.600

PERQ3 0.938 0.947 0.952 0.978 1.004 1.009 1.018

PERV BY

PERV1 1.607 1.615 1.620 1.643 1.667 1.671 1.680

PERV2 1.809 1.820 1.826 1.856 1.885 1.891 1.902

CUEX WITH

ACSI 0.591 0.596 0.598 0.612 0.626 0.629 0.634

PERQ WITH

ACSI 0.951 0.952 0.953 0.957 0.961 0.962 0.963

CUEX 0.679 0.683 0.686 0.698 0.711 0.713 0.718

PERV WITH

ACSI 0.865 0.867 0.868 0.875 0.881 0.882 0.885

CUEX 0.501 0.507 0.510 0.526 0.542 0.545 0.551

PERQ 0.755 0.759 0.761 0.770 0.779 0.780 0.784

We will next show how these results can be used to calculate all the statistics and tests explained in the article.

## Factor correlation (point estimate)

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

The advantage of fixing the variance of latent variables to 1 is that the estimated factor covariance are correlations that can be interpreted directly without any further calculation.

Table 31 Factor correlation estimates from Mplus

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .612 | 1.000 |  |  |
| PERQ | .957 | .698 | 1.000 |  |
| PERV | .875 | .526 | . 770 | 1.000 |

### Problems / Limitations

The point estimate provides only limited information about the parameter. Particularly, a single point estimate does not tell us anything about how certain we are about the estimate. A better alternative is an interval estimate in the form of a 95% confidence interval, which gives information on the maximum and minimum values of the parameter when the assumptions are met at a given confidence level.

## Factor correlation (whether the confidence interval includes 1)

### How often is it used?

AMJ 0%, JAP 0%, ORM 5.0%

### How to obtain

If you specify the following, Mplus presents the confidence interval of the estimates.

OUTPUT:

CINTERVAL;

Several confidence intervals are presented. If the usual significance level of 5% is applied, the lower limit is indicated in the lower 2.5% column, and the upper limit is indicated in the upper 2.5% column. That is, the following table can be obtained.

Table 32 Confidence intervals for correlations obtained from Mplus

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | [.596,.629] | 1.000 |  |  |
| PERQ | [.952,.962] | [.683,.713] | 1.000 |  |
| PERV | [.867,.882] | [.507,.545] | [.759,.780] | 1.000 |

### Problems / Limitations

There is no problem with this technique itself, but the problem lies in the way we have used this technique this far. When evaluating discriminant validity, we have examined whether the interval estimates of factor correlation include one (i.e., perfect correlation). That is, if the maximum value of the confidence interval is less than 1, it is determined that there is no discriminant validity problem. This is problematic because almost all data will meet these criteria as long as the sample size is large enough. For example, in the above example, the factor correlation between ACSI-PERQ is very high, but its confidence interval does not include 1.

## Techniques using model fit indices: no comparison

### How often is it used?

AMJ 11.1%, JAP 1.4%, ORM 0%

### How to obtain

The values of the model fit indices are automatically calculated in Mplus. For example, this case the fit indices are = 1614.286, p = 0.0000, TLI 970, CFI .979, and RMSEA .063. The value shows that the model does not fit exactly. While there are SEM model evaluation guidelines that provide cutoffs for the other indices and our values would be considered acceptable against these cutoffs, we nevertheless suggest that researchers diagnose their models to understand the source of misfit before declaring misfit acceptable (Kline, 2011, Chapter 8). However, applying this technique makes no indication of any problem in the discriminant validity of these data.

### Problems / Limitations

The fit of the proposed model has nothing to do with the discriminant validity. Assessing discriminant validity requires a well-fitting model, but the model fit itself does not inform us about discriminant validity. To assess discriminant validity using model fit indices, a comparison with other alternative models is needed. The question is which alternative model to compare.

## Techniques using model fit indices: compared to nested models with fewer factors

### How often is it used?

AMJ 29.6%, JAP 58.9%, ORM 25.0%

As far as we know, there are no guidelines-type article that recommends the use of this technique for evaluating discriminant validity. Surprisingly, however, this technique is the most commonly used technique in applied psychology.

### How to obtain

Because there is no authoritative source, this technique is being applied in a wide variety of ways. A typical method is as follows. Suppose the proposed model is composed of N factors. Then, we can construct (N - 1) -factor models, (N - 2)-factor models … and a 1-factor model by merging some of the factors into one. This technique compares the fit indices of all these (or some arbitrarily selected) alternative models with the originally proposed models.

In our ACSI example, the factor correlation between ACSI and PERQ is the highest, so you can compare an alternative three-factor model that combines the two factors into one factor with the originally proposed model. (Of course, it is also possible to review comparisons with all possible three-factor, two-factor, and one-factor models.) In other words, we construct the following model in Mplus:

MODEL:

! Free the first loadings with stars (\*)

ACSIPERQ BY ACSI1\* ACSI2 ACSI3 PERQ1 PERQ2 PERQ3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERV BY PERV1\* PERV2;

! Constrain the factor variances

ACSIPERQ@1;

CUEX@1;

PERV@1;

Table 33 Model fit indides for the comparison of three and four factor models

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. Alternative 3-factor model | .961 | .948 | .083 | 41 | 2972.521 | .000 |
| b. Proposed 4-factor model | .979 | .970 | .063 | 38 | 1614.286 | .000 |
| Difference (a - b) |  |  |  | 3 | 1358.235 | .000 |

The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

### Problems / Limitations

Notice that the difference in degrees of freedom between the proposed model and the alternative model is three. Originally, our interest was a high correlation between ACSI and PERQ, but merging the two factors into one adds additional constraints. In other words, the above model is equivalent to the original model with the following three constraints.

1. The correlation between ACSI and PERQ (.957) is 1.
2. The correlation between ACSI and CUEX (.612) and the correlation between PERQ and CUEX (.698) are equal.
3. The correlation between ACSI and PERV (.875) and the correlation between PERQ and PERV (. 770) are equal.

In other words, the above model is mathematically equivalent to the following model:

MODEL:

! Free the first loadings with stars (\*)

ACSI BY ACSI1\* ACSI2 ACSI3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERQ BY PERQ1\* PERQ2 PERQ3;

PERV BY PERV1\* PERV2;

! Constrain the factor variances

ACSI@1;

CUEX@1;

PERQ@1;

PERV@1;

! Constrain correlations

ACSI WITH PERQ@1;

ACSI WITH CUEX (a);

PERQ WITH CUEX (a);

ACSI WITH PERV (b);

PERQ WITH PERV (b);

Of these, only the first constraint is truly relevant to discriminant validity, and a strategy that focuses only on the necessary constraints is needed.

## Techniques using model fit indices: comparison against model with correlation fixed at 1

### How often is it used?

AMJ 14.8%, JAP 1.4%, ORM 10.0%

### How to obtain

For all possible latent variable pairs, the model with the correlation fixed at 1 is compared with the original model. Here, we present only the model with the correlation between ACSI and PERQ is constrained to 1. That is, we compare the following model with the original model.

MODEL:

ACSI BY ACSI1\* ACSI2 ACSI3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERQ BY PERQ1\* PERQ2 PERQ3;

PERV BY PERV1\* PERV2;

ACSI@1;

CUEX@1;

PERQ@1;

PERV@1;

! Added a constraint on a correlation

ACSI WITH PERQ@1;

Table 34 Model fit indices for nested model test for perfect correlation

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. A model with a fixed correlation of 1 | .974 | .963 | .069 | 39 | 1999.773 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.286 | .000 |
| Difference (a - b) |  |  |  | 1 | 385.487 | .000 |

Although the correlation between ACSI and PERQ is very high at .957, the model with a fixed correlation of 1 has poorer fit indices than the original model.

### Problems / Limitations

There are no logical flaws in this technique. However, there is a practical problem that almost all data pass the criteria as long as the sample size is large enough because correlations are rarely exactly 1. Thus, applied researchers have preferred a technique that require more difficult-to-pass criteria.

## AVE: compared with the square of factor correlation

### How often is it used?

AMJ 7.4%, JAP 5.5%, ORM 5.0%

Although this technique is not used very often among organizational researchers, it is a standard technique for evaluating discriminant validity in many other business disciplines, such as marketing.

### How to obtain

The original formula of Average Variance Extracted (AVE), proposed by Fornell and Larcker (1981) is:

where is the standardized loading of indicator . Because of standardization, this can be simplified to be the mean of squared factor loadings.

For example, let's calculate ACSI’s AVE using this simpler formula:

If we use the original formula and the unstandardized coefficients, the resulting AVE will be different:

As can be seen from the above calculations, the two versions of the AVE formula usually yield close values, but are not mathematically equivalent. Instead of a mathematical proof of how the two formulas differ, we present a simple analogy. For example, assume that there are three values , , and . The original formula is to calculate , and the formula using the standardized coefficients is to calculate ++) . For the same reason, the value obtained by applying the standardized coefficients to the original formula of AVE is different from the value obtained by applying the unstandardized coefficients to the same formula. Therefore, when presenting the AVE value, we should specify whether standardized or unstandardized coefficients are used. However, note that the interpretation of AVE as variance explained is only valid if the latent variables were scaled to unit variances (which would be automatically the case in fully standardized estimates).

We will now explain how the AVE values are used in the The Fornell-Larcker criterion for assessing discriminant validity. The table below shows the AVE values using unstandardized coefficients on the diagonal (in italics), the previously reported factor correlations on the lower-triangle and the squares of the factor correlations are shown on the upper triangle (e.g.,.

Table 35 Comparison of AVEs against squared factor correlation estimates

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .375 | .916 | .766 |
| CUEX | .612 | *.444* | .487 | .277 |
| PERQ | .957 | .698 | *.632* | .593 |
| PERV | .875 | .526 | . 770 | *.773* |

The Fornell-Larcker criterion for assessing discriminant validity is that for every pair of latent variables, the square of the factor correlation must be less than the AVE values of both latent variables. The acronym AVE/SV comes from the fact that the square of the factor correlation is also called shared variance. The above example fails this criterion for three pairs of latent variables: 1) The shared variance between ACSI and PERQ is .916, which is larger than ACSI's AVE value of .713 and PERQ's AVE value of .632. 2) The shared variance between ACSI and PERV is .766, which is larger than the ACSI’s AVE value of .713. 3) The shared variance between CUEX and PERQ is .487, which is greater than the CUEX’s AVE value of .444.

### Problems / Limitations

Few methodological studies have seriously considered this technique, and the logical problems of this technique are almost unknown. Essentially, AVE is a weighted average of item-level reliabilities. While factor correlation is related to discriminant validity, as a comparison point, AVE has little relevance. Moreover, given methodological literature’s lack of attention to this technique, it is often is misapplied in a variety of ways. The most common misuse is that AVE is compared to a value other than the square of the factor correlation.

## AVE: compared with the square of scale score correlation

### How often is it used?

AMJ 3.7%, JAP 1.4%, ORM 0%

Many users are more familiar with the process of obtaining scale score correlations than factor correlations. Therefore, unlike the original proposal of Fornell and Larcker (1981a), studies comparing AVE to the square of the scale score correlation are often found.

### How to obtain

AVE values are shown on the diagonals (in italics). The previously reported scale score correlation (Table 2) is shown in the sub-diagonals. The square of the scale score correlation is displayed in the super-diagonals (e.g.,.

Table 36 Comparison of AVEs against squared scale score correlations

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | *.713* | .235 | .667 | .585 |
| CUEX | .485 | *.444* | .303 | .163 |
| PERQ | .817 | .550 | *.632* | .416 |
| PERV | .765 | .404 | .645 | *.773* |

The Fornell-Larcker criterion was violated in only one case. The square of scale score correlation between ACSI and PERQ is .667, which is larger than PERQ's AVE value of .632.

### Problems / Limitations

It is clear that this technique is misuse, so much explanation is not necessary.

## Techniques using model fit indices: comparison against model with correlation fixed at cutoff point less than 1

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

The fact that a correlation is not exactly 1 may be a necessary condition for discriminant validity, but it is difficult to consider it a sufficient condition. In other words, a more realistic test would test against a high but not necessarily perfect correlation. This is the idea in using references values below 1. Which cutoff to use is a matter of subjective judgment, but we can consider candidates such as .85, .9, or .95. This test makes sense only if the correlation estimate is below the the cutoff; if it does not, the discriminant validity test should be considered as failed. To demonstrate this test, we compare the correlation between ACSI and PERV against a fixed cutoff of .96, which in this case is just an arbitrarily chosen cutoff that is greater than the estimated correlation of .957. That is, we compare the following model with the original model.

MODEL:

ACSI BY ACSI1\* ACSI2 ACSI3;

CUEX BY CUEX1\* CUEX2 CUEX3;

PERQ BY PERQ1\* PERQ2 PERQ3;

PERV BY PERV1\* PERV2;

ACSI@1;

CUEX@1;

PERQ@1;

PERV@1;

! Added a constraint on a correlation

ACSI WITH PERQ@.96;

Table 37 Model fit indices for nested model test for correlation constrained to a cutoff other than 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | CFI | TLI(NNFI) | RMSEA | df |  | p |
| a. A model with a fixed correlation of .96 | .979 | .970 | .062 | 39 | 1615.895 | .000 |
| b. Original model | .979 | .970 | .063 | 38 | 1614.286 | .000 |
| Difference (a - b) |  |  |  | 1 | 1.609 | .205 |

If the reference point is .96, the correlation between ACSI and PERV is not significantly less than that, so we can assess that there is a problem in the discriminant validity.

### Problems / Limitations

This technique has the advantage of being more flexible than the previous techniques, but there are also limitations. There is room for criticism that the user must determine the reference value, which is an arbitrary choice. Mplus does not provide a feature to do several model comparisons in one analysis, and specifying the constraints one at a time is tedious.

## Structure coefficients (obtained from CFA)

### How often is it used?

AMJ 7.4%, JAP 0%, ORM 0%

### How to obtain

A typical CFA is an independent clusters (IC) model, where each item loads on just one factor and the loading value with other factors is zero. In the case of a CFA model specification, the term loading invariably refers to the factor pattern coefficients because the factor structure coefficients are not model parameters that are estimated, but are something that can be calculated post-estimation.

While Mplus does not provide structure coefficients directly, we explain how to obtain them. Factor structure coefficients are the matrix multiplication of factor pattern coefficients and factor correlations. That is, the (i, j) th element of the third matrix is obtained by multiplying the (i, k) element of the first matrix by the element (k, j) of the second matrix, and summing it over all k. For example, .567 in factor structure coefficients is obtained by the following calculation. **.567** = **.926** \* **.612**+ **.000** \* **1.000** + **.000** \* **.698** + **.000** \* **.526**. One convenient way to calculate this matrix product is to use Microsoft Excel’s MMULT worksheet function.

We will now demonstrate the calculation. Let's determine the cross-loading according to some rules discussed above. First, absolute comparison. Let's choose .4 as the cutoff point. There are cross-loadings in all items except CUEX3. Second, row comparison. There is no cross-loading on any item. Third, column comparison. Cross-loadings can be found in five items: ACSI2, ACSI3, CUEX3, PERQ2, and PERQ3.

Table 38 Calculating factor structure coefficients from Mplus output

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A.** Factor pattern coefficients | | | | **B.** Factor correlations | | | | **C.** Factor structure coefficients | | | |
| ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV | ACSI | CUEX | PERQ | PERV |
| **.926** | **.000** | **.000** | **.000** | 1.000 | **.612** | .957 | .875 | 0.926 | **0.567** | 0.886 | 0.810 |
| .831 | .000 | .000 | .000 | .612 | **1.000** | .698 | .526 | 0.831 | 0.509 | 0.795 | 0.727 |
| .768 | .000 | .000 | .000 | .957 | **.698** | 1.000 | .770 | 0.768 | 0.470 | 0.735 | 0.672 |
| .000 | .753 | .000 | .000 | .875 | **.526** | . 770 | 1.000 | 0.461 | 0.753 | 0.526 | 0.396 |
| .000 | .749 | .000 | .000 |  |  |  |  | 0.458 | 0.749 | 0.523 | 0.394 |
| .000 | .451 | .000 | .000 |  |  |  |  | 0.276 | 0.451 | 0.315 | 0.237 |
| .000 | .000 | .901 | .000 |  |  |  |  | 0.862 | 0.629 | 0.901 | 0.694 |
| .000 | .000 | .868 | .000 |  |  |  |  | 0.831 | 0.606 | 0.868 | 0.668 |
| .000 | .000 | .575 | .000 |  |  |  |  | 0.550 | 0.401 | 0.575 | 0.443 |
| .000 | .000 | .000 | .913 |  |  |  |  | 0.799 | 0.480 | 0.703 | 0.913 |
| .000 | .000 | .000 | .844 |  |  |  |  | 0.739 | 0.444 | 0.650 | 0.844 |

### Problems / Limitations

As discussed earlier, checking for cross-loadings is not a technique that closely matches the definition of discriminant validity.

## CICFA(.9): the proposed technique

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

There are no studies that formally proposed this technique.

### How to obtain

The method of obtaining the upper limit of factor correlation in Mplus has been described above (i.e.,Table 32). The difference between the existing technique and CICFA (.9) is what this upper limit is compared to. The existing technique evaluates that there is a problem with discriminant validity if the upper limit is larger than 1. Our concern is not whether the correlation is exactly 1, but whether it is a sufficiently low number of 1, so the existing technique does not fit well with the definition of discriminant validity. We suggest using a number less than 1 as a cutoff. CICFA (.9) evaluates that there is a problem with discriminant validity if the upper limit is larger than .9. The upper limit of the correlation between ACSI-PERQ is .962, which does not pass this criterion.

### Problems / Limitations

The choice of a cutoff is inevitable for a dichotomous (i.e., yes or no) decision, but there is room for criticism that any cutoff is arbitrary. More research is needed as to which cutoff is best, and different cutoffs may be considered depending on the purpose of the study (e.g., CICFA(.85)).

All techniques using R

The R tutorial is presented as an R notebook, which shows the full R syntax and output, that has been compiled as a document. The output lines start with a double hash (##). All other lines are R code. . We will use the lavaan (Rosseel, 2012), Psych (Revelle, 2015) and SEMTools (Jorgensen, Pornprasertmanit, Schoemann, & Rosseel, 2018) packages for the analyses.The R file can be downloaded at <https://github.com/eunscho/DiscriminantValidityTutorial>

## R syntax for setting up the data

library(lavaan)

## This is lavaan 0.6-3

## lavaan is BETA software! Please report any bugs.

library(semTools)

##

## ##########################################################################

## This is semTools 0.5-1

## All users of R (or SEM) are invited to submit functions or ideas for functions.

## ##########################################################################

library(psych)

##   
## Attaching package: 'psych'

## The following object is masked from 'package:semTools':  
##   
## skew

## The following object is masked from 'package:lavaan':  
##   
## cor2cov

# Set up the covariance data  
  
ECSICovData <- matrix(NA,11,11)  
rownames(ECSICovData) <-   
 colnames(ECSICovData) <- c("ACSI1", "ACSI2", "ACSI3", "CUEX1", "CUEX2",   
 "CUEX3", "PERQ1", "PERQ2", "PERQ3", "PERV1",  
 "PERV2")  
  
ECSICovData[upper.tri(ECSICovData, diag = TRUE)] <-  
 c(4.00,  
 3.23, 4.41,  
 2.66, 2.67, 3.61,  
 1.81, 1.50, 1.56, 4.41,  
 1.85, 1.62, 1.63, 2.63, 4.84,  
 1.24, 1.16, 1.09, 1.55, 1.72, 5.29,  
 3.04, 2.83, 2.35, 2.07, 1.96, 1.31, 3.61,  
 2.81, 2.57, 2.19, 1.55, 1.74, 1.12, 2.67, 3.24,  
 1.73, 1.64, 1.32, 0.89, 1.05, 1.41, 1.62, 1.62, 2.89,  
 2.66, 2.49, 2.12, 1.40, 1.43, 0.95, 2.22, 2.01, 1.25, 3.24,  
 2.99, 2.86, 2.42, 1.52, 1.50, 1.01, 2.34, 2.14, 1.31, 3.05, 4.84)  
  
ECSICovData[lower.tri(ECSICovData)] <-  
 t(ECSICovData)[lower.tri(ECSICovData)]  
  
N <- 10417

## Scale score correlation

### How often is it used?

AMJ 25.9%, JAP 11.0%, ORM 55.0%

This is probably the most easily understood among the various techniques, so it is commonly used.

### How to obtain

Scales scores are typically the unweighted sum or mean of observed item scores. We use matrix algebra to calculate the scale score correlation matrix directly from the indicator correlation matrix.

# A matrix that defines how the indicators are summed  
  
W <- diag(4)[c(1,1,1,2,2,2,3,3,3,4,4),]  
rownames(W) <- rownames(ECSICovData)  
colnames(W) <- c("ACSI","CUEX","PERQ","PERV")  
  
scaleScoreCorrelations <- cov2cor(t(W)%\*%ECSICovData%\*%W)  
scaleScoreCorrelations

## ACSI CUEX PERQ PERV  
## ACSI 1.0000000 0.4858390 0.8170729 0.7644835  
## CUEX 0.4858390 1.0000000 0.5497168 0.4041151  
## PERQ 0.8170729 0.5497168 1.0000000 0.6445571  
## PERV 0.7644835 0.4041151 0.6445571 1.0000000

### Problems / Limitations

Scale score correlation is a function of 'true correlation' and scale score reliability. That is, a low scale score correlation may be due to a low true correlation, or a low reliability. Therefore, these correlations alone do not allow any conclusions about discriminant validity to be drawn.

## Disattenuated correlation using tau-equivalent reliability

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

As mentioned above, scale score correlation is a function of 'true correlation' and scale score reliability. Therefore, the true correlation can be estimated by adjusting the scale score correlation to account for its unreliability. The estimate of true correlation obtained in this way is called disattenuated correlation. The effectiveness of this technique depends on a) how accurate the reliability estimates are used and b) to the extent that the indicators are contaminated by non-random measurement error. In typical applications, non-random measurement error is assumed to not exist.

There are many ways to estimate scale score reliability, but the most commonly used is tau-equivalent reliability. This reliability coefficient is often incorrectly known as Cronbach's alpha. Kuder and Richardson (1937), not Cronbach (1951), developed this coefficient first (see Cho & Kim, 2015). Tau-equivalent reliability can be calculated for example by using the psych package (Revelle, 2015). After the reliability estimates have been calculated, disattenuated correlationcan be calculated as follows:

##### Disattenuated correlation using tau-equivalent reliability #####  
  
tauEquivRel <- c(alpha(ECSICovData[1:3,1:3])$total$raw\_alpha,  
 alpha(ECSICovData[4:6,4:6])$total$raw\_alpha,  
 alpha(ECSICovData[7:9,7:9])$total$raw\_alpha,  
 alpha(ECSICovData[10:11,10:11])$total$raw\_alpha)

## Warning in matrix(unlist(drop.item), ncol = 10, byrow = TRUE): data length  
## [16] is not a sub-multiple or multiple of the number of columns [10]

disattenuatedCorrelations <- scaleScoreCorrelations/  
 sqrt(tauEquivRel%o%tauEquivRel)  
  
diag(disattenuatedCorrelations) <- 1  
  
disattenuatedCorrelations

## ACSI CUEX PERQ PERV  
## ACSI 1.0000000 0.6313365 0.9597941 0.8779577  
## CUEX 0.6313365 1.0000000 0.7394866 0.5314770  
## PERQ 0.9597941 0.7394866 1.0000000 0.7662838  
## PERV 0.8779577 0.5314770 0.7662838 1.0000000

### Problems / Limitations

Tau-equivalent reliability is widely misunderstood and misused (Cho, 2016; Cho & Kim, 2015). Perhaps the most common misconception is that it is a general reliability coefficient that can be applied to all data. Tau-equivalent reliability is based on the assumption that each variable is influenced by the true score to the same extent, which implies equal covariances as shown in Table 5. For example, to satisfy tau-equivalence, the covariances between ACSI1, ACSI2, and ACSI3 that make up ACSI must all have the same value. The covariance of ACSI1-ACSI3 is 2.66 and the covariance of ACSI2-ACSI3 is 2.67, so it appears that tau-equivalence is met. However, the covariance of ACSI1-ACSI2 is 3.23, which differs greatly from the other two values and thus does not meet tau-equivalence. Therefore, a technique that does not depend on the assumption of tau-equivalence is needed.

Table 39 Examples of parallel, tau-equivalent, and congeneric covariances

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A. Parallel data | | | | B. Tau-equivalent data | | | | C. Congeneric data | | | |
|  | X1 | X2 | X3 |  | X1 | X2 | X3 |  | X1 | X2 | X3 |
| X1 | 10 | 4 | 4 | X1 | 10 | 4 | 4 | X1 | 10 | 2 | 3 |
| X2 | 4 | 10 | 4 | X2 | 4 | 9 | 4 | X2 | 2 | 9 | 6 |
| X3 | 4 | 4 | 10 | X3 | 4 | 4 | 11 | X3 | 3 | 6 | 11 |
| All three values are correct | | | | and are correct | | | | Only is correct  , | | | |

## Disattenuated correlation using parallel reliability (so called HTMT)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

This technique is rarely used by organizational researchers yet. However, its popularity has exploded in other related fields after Henseler, Ringle and Sarstedt (2015) has introduced this technique under the name Heterotrait-Monotrait (HTMT).

### How to obtain

Parallel reliability is often mistakenly referred to as standardized alpha, which gives a misleading impression that it is a variant of tau-equivalent reliability (i.e., alpha). The two reliability coefficients are based on different assumptions and are independent formulas. As with tau-equivalent reliability, parallel reliability is available in the psych package (Revelle, 2015).

##### Disattenuated correlation using parallel reliability (HTMT) #####  
  
  
scaleScoreCorrelationsStd <- cov2cor(t(W)%\*%cov2cor(ECSICovData)%\*%W)  
  
parallelRel <- c(alpha(ECSICovData[1:3,1:3])$total$std.alpha,  
 alpha(ECSICovData[4:6,4:6])$total$std.alpha,  
 alpha(ECSICovData[7:9,7:9])$total$std.alpha,  
 alpha(ECSICovData[10:11,10:11])$total$std.alpha)

## Warning in matrix(unlist(drop.item), ncol = 10, byrow = TRUE): data length  
## [16] is not a sub-multiple or multiple of the number of columns [10]

# We use the sum of standardized indicators  
  
disattenuatedCorrelations <- scaleScoreCorrelationsStd/  
 sqrt(parallelRel%o%parallelRel)  
  
diag(disattenuatedCorrelations) <- 1  
  
disattenuatedCorrelations

## ACSI CUEX PERQ PERV  
## ACSI 1.0000000 0.6335125 0.9548449 0.8761793  
## CUEX 0.6335125 1.0000000 0.7364370 0.5336834  
## PERQ 0.9548449 0.7364370 1.0000000 0.7645405  
## PERV 0.8761793 0.5336834 0.7645405 1.0000000

Disattenuated correlation using parallel reliability can be also calculated using the htmt function of the semTools package (Jorgensen et al., 2018). This requires the specification (but not estimation) of a CFA model using lavaan syntax.

CFAmodel <- "ACSI =~ ACSI1 + ACSI2 + ACSI3  
CUEX =~ CUEX1 + CUEX2 + CUEX3  
PERQ =~ PERQ1 + PERQ2 + PERQ3  
PERV =~ PERV1 + PERV2"  
  
htmt(CFAmodel, sample.cov = ECSICovData)

## ACSI CUEX PERQ PERV   
## ACSI 1.000   
## CUEX 0.634 1.000   
## PERQ 0.955 0.736 1.000   
## PERV 0.876 0.534 0.765 1.000

### Problems / Limitations

Parallel reliability is less accurate than tau-equivalent reliability. Therefore, disattenuated correlation using parallel reliability is less effective than disattenuated correlation using tau-equivalent reliability for assessing discriminant validity. In addition to tau-equivalence, parallel reliability also requires the assumption that the variance of each item is identical (i.e., the condition of being parallel). In other words, it is more difficult to satisfy the assumption of parallel reliability than to satisfy that of tau-equivalent reliability. Parallel reliability has no computational advantage over tau-equivalent reliability. Therefore, the use of disattenuated correlation using parallel reliability (i.e., HTMT) for the assessment of discriminant validity is difficult to justify.

## Disattenuated correlation using congeneric reliability

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

Congeneric reliability is more accurate than tau-equivalent reliability but less frequently used. This coefficient is often referred to as composite reliability or McDonald's omega. Jöreskog (1971) developed this coefficient first and named its measurement model a congeneric model. The likely reason for the lower adoption of this coefficient compared to the previously presented reliability coefficients is that this coefficient cannot be calculated directly from the data, but requires the estimation of a factor analysis model, typically a CFA. Congeneric reliabilities can be obtained using the reliability function of the semTools package (Jorgensen et al., 2018).

CFAest <- cfa(CFAmodel, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling)  
  
# Use the semTools reliability function  
congenericRel <- reliability(CFAest)["omega",1:4]  
  
disattenuatedCorrelations <- scaleScoreCorrelations/  
 sqrt(congenericRel%o%congenericRel)  
  
diag(disattenuatedCorrelations) <- 1  
  
disattenuatedCorrelations

## ACSI CUEX PERQ PERV  
## ACSI 1.0000000 0.6235469 0.9484115 0.8758320  
## CUEX 0.6235469 1.0000000 0.7225404 0.5242577  
## PERQ 0.9484115 0.7225404 1.0000000 0.7562414  
## PERV 0.8758320 0.5242577 0.7562414 1.0000000

### Problems / Limitations

Disattenuated correlation using congeneric reliability is more general than the previous methods. Generally, the disattenuatio techniques are commonly recommended as alternatives for researchers who have difficulty using SEM software. However, disattenuated correlation technique itself has problems. First, a correlation coefficient that is greater than 1 or less than -1 may be derived. Second, the correlation estimates can be less precise and the calculation process has more steps than when directly estimating factor correlations based on CFA or SEM thus leaving more room for errors. Third, although it is possible to obtain the confidence interval of disattenuated correlation, doing this correctly using equiations is complicated. Another alternative is to apply boostrapping, but this often requires a bit of programming of the statistical software and can be too computationally intensive to be a practical alternative.

## Cross-loadings (obtained from exploratory factor analysis)

### How often is it used?

AMJ 3.7%, JAP 0%, ORM 15.0%

### How to obtain

The inspection of cross-loadings is sometimes suggested as a way for assessing discrimination validity. However, the term cross-loading has at least two different meanings in the literature that need to be explicitly explained.

1. What is a loading? That is, is it a pattern coefficient or a structure coefficient?
2. What is a *cross*-loading? That is, does the determination of the existence of a problematic cross-loadings require absolute comparisons (e.g., cutoff point) or relative comparisons (e.g., other coefficient value)?

The proper application of an exploratory factor analysis (EFA) is a complex subject in itself, so we provide just a brief demonstration instead of fully explaining the analysis. We selected minres likelihood as the factor extraction method, Varimax as the orthogonal rotation, and Promax as the oblique rotation using the fa function from the psych package (Revelle, 2015). The number of factors was determined to be two when determined according to the commonly used criterion that the eigenvalues of the factors should all be greater than one[[2]](#footnote-2).

First, let us consider the first question. Factor loadings represent pattern coefficients or structure coefficients depending on the context. The pattern coefficients indicate how the item value changes when the factor’s (unobserved) value changes by one unit holding other factors constant, which is similar to the coefficient in the regression analysis. The structure coefficient is the correlation between items and factors.

Exploratory factor analysis results will generally need to be rotated to make them more interpretable. We start by considering the orthogonal rotation. The word orthogonal is a geometric term, and its corresponding statistical term is ‘being uncorrelated with each other’. That is, the correlation between the two factors is fixed as follows.

Table 40 Factor correlation matrix after orthogonal rotation

|  | Factor 1 | Factor 2 |
| --- | --- | --- |
| Factor 1 | 1 | 0 |
| Factor 2 | 0 | 1 |

The factor structure matrix is the matrix product of the factor pattern matrix and the factor correlation matrix. Because the correlation matrix is an identity matrix (i.e., a matrix composed of diagonal elements of 1 and non-diagonal elements of 0) in orthogonal rotation, the structure matrix and the pattern matrix are identical. The following is the result of Varimax rotation of the example

# Two factor solution, varimax rotation  
  
EFAest <- fa(ECSICovData, nfactors = 2, rotate = "varimax")  
  
# Pattern coefficients are printed out by default  
EFAest

## Factor Analysis using method = minres  
## Call: fa(r = ECSICovData, nfactors = 2, rotate = "varimax")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## MR1 MR2 h2 u2 com  
## ACSI1 0.86 0.33 0.85 0.15 1.3  
## ACSI2 0.80 0.24 0.70 0.30 1.2  
## ACSI3 0.70 0.32 0.60 0.40 1.4  
## CUEX1 0.24 0.68 0.52 0.48 1.2  
## CUEX2 0.22 0.71 0.56 0.44 1.2  
## CUEX3 0.17 0.43 0.21 0.79 1.3  
## PERQ1 0.73 0.47 0.75 0.25 1.7  
## PERQ2 0.73 0.40 0.69 0.31 1.5  
## PERQ3 0.47 0.31 0.31 0.69 1.7  
## PERV1 0.79 0.22 0.68 0.32 1.2  
## PERV2 0.74 0.17 0.58 0.42 1.1  
##   
## MR1 MR2  
## SS loadings 4.47 1.98  
## Proportion Var 0.41 0.18  
## Cumulative Var 0.41 0.59  
## Proportion Explained 0.69 0.31  
## Cumulative Proportion 0.69 1.00  
##   
## Mean item complexity = 1.4  
## Test of the hypothesis that 2 factors are sufficient.  
##   
## The degrees of freedom for the null model are 55 and the objective function was 7.2  
## The degrees of freedom for the model are 34 and the objective function was 0.46   
##   
## The root mean square of the residuals (RMSR) is 0.04   
## The df corrected root mean square of the residuals is 0.05   
##   
## Fit based upon off diagonal values = 0.99  
## Measures of factor score adequacy   
## MR1 MR2  
## Correlation of (regression) scores with factors 0.94 0.83  
## Multiple R square of scores with factors 0.89 0.69  
## Minimum correlation of possible factor scores 0.77 0.37

# Structure coefficients equal pattern coefficients  
EFAest$Structure

##   
## Loadings:  
## MR1 MR2   
## ACSI1 0.861 0.327  
## ACSI2 0.800 0.241  
## ACSI3 0.704 0.316  
## CUEX1 0.240 0.681  
## CUEX2 0.224 0.711  
## CUEX3 0.172 0.430  
## PERQ1 0.726 0.474  
## PERQ2 0.729 0.395  
## PERQ3 0.467 0.310  
## PERV1 0.794 0.223  
## PERV2 0.739 0.170  
##   
## MR1 MR2  
## SS loadings 4.468 1.976  
## Proportion Var 0.406 0.180  
## Cumulative Var 0.406 0.586

While this example would suggest that there is not much difference between the pattern and structure matrices, it would be a mistake to assume so. Orthogonal rotation is based on an unrealistic assumption that factors are uncorrelated and should thus be avoided in research that aims to study correlations between constructs. The following is the result of Promax rotation of the example.

# Two factor solution, promax rotation  
  
EFAest <- fa(ECSICovData, nfactors = 2, rotate = "promax")

## Loading required namespace: GPArotation

# Pattern coefficients are printed out by default  
EFAest

## Factor Analysis using method = minres  
## Call: fa(r = ECSICovData, nfactors = 2, rotate = "promax")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## MR1 MR2 h2 u2 com  
## ACSI1 0.91 0.01 0.85 0.15 1.0  
## ACSI2 0.88 -0.07 0.70 0.30 1.0  
## ACSI3 0.73 0.07 0.60 0.40 1.0  
## CUEX1 -0.02 0.73 0.52 0.48 1.0  
## CUEX2 -0.05 0.78 0.56 0.44 1.0  
## CUEX3 0.01 0.45 0.21 0.79 1.0  
## PERQ1 0.68 0.25 0.75 0.25 1.3  
## PERQ2 0.72 0.15 0.69 0.31 1.1  
## PERQ3 0.43 0.17 0.31 0.69 1.3  
## PERV1 0.88 -0.09 0.68 0.32 1.0  
## PERV2 0.84 -0.13 0.58 0.42 1.0  
##   
## MR1 MR2  
## SS loadings 4.86 1.58  
## Proportion Var 0.44 0.14  
## Cumulative Var 0.44 0.59  
## Proportion Explained 0.75 0.25  
## Cumulative Proportion 0.75 1.00  
##   
## With factor correlations of   
## MR1 MR2  
## MR1 1.00 0.65  
## MR2 0.65 1.00  
##   
## Mean item complexity = 1.1  
## Test of the hypothesis that 2 factors are sufficient.  
##   
## The degrees of freedom for the null model are 55 and the objective function was 7.2  
## The degrees of freedom for the model are 34 and the objective function was 0.46   
##   
## The root mean square of the residuals (RMSR) is 0.04   
## The df corrected root mean square of the residuals is 0.05   
##   
## Fit based upon off diagonal values = 0.99  
## Measures of factor score adequacy   
## MR1 MR2  
## Correlation of (regression) scores with factors 0.97 0.89  
## Multiple R square of scores with factors 0.95 0.80  
## Minimum correlation of possible factor scores 0.89 0.59

# Structure coefficients differ from pattern coefficients  
EFAest$Structure

##   
## Loadings:  
## MR1 MR2   
## ACSI1 0.921 0.608  
## ACSI2 0.834 0.506  
## ACSI3 0.770 0.543  
## CUEX1 0.463 0.722  
## CUEX2 0.457 0.744  
## CUEX3 0.311 0.463  
## PERQ1 0.846 0.699  
## PERQ2 0.821 0.626  
## PERQ3 0.545 0.453  
## PERV1 0.822 0.487  
## PERV2 0.752 0.418  
##   
## MR1 MR2  
## SS loadings 5.584 3.709  
## Proportion Var 0.508 0.337  
## Cumulative Var 0.508 0.845

When oblique rotation is used, the meaning of the term “factor loading” is ambiguous as it can refer to either structure coefficients or pattern coefficients, which are clearly different quantities as the example shows. It is also worth noting that in the above correlation matrix, the correlation between the two factors is very high at .65. That is, the assumption that the two factors are not correlated with each other is far from reality and the results of orthogonal rotation can be very misleading.

Now, let's consider the second question. In other words, what value should the loading be greater than to be considered a problematic 'cross-loading'? The first method to detect if cross-loading exists is absolute comparison. If the absolute value of the coefficient between an item and a factor is higher than an arbitrary cutoff point (e.g., .3, .4, .5), then the item is considered to be 'loaded' on the factor. If an item is 'loaded' on more than one factor, the item is considered to be 'cross-loaded'. For example, let's say the cutoff point is .4, which is a fairly commonly used cutoff. In the above structure matrix, all items are cross-loaded except for CUEX2 and CUEX3. CUEX3 is not loaded on any factor. Cross-loadings are less common in the pattern matrix and are observed only in ACSI1 and ACSI2. The problem with this method is that the choice of an cutoff is arbitrary and changing the cutoff changes the judgment of cross-loadings substantially. For example, if the cutoff point is .5, there is no cross-loading in the above pattern matrix, but ACSI2, ACSI3, and CUEX3 are not loaded on any factor.

The second method is relative comparison. In this comparison, an item is 'loaded' on a factor which it has highest loading on among all the factors. These coefficient values are shown in boldface in the table above. We can think of two rules related to cross-loadings. The first rule is what we call row comparison (Henseler et al., 2015). That is, 'for *an item*, if the absolute value of the coefficient between the item and the loaded factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between the item and any unloaded factors (e.g., ACSI-Factor2), the item is cross-loaded. ' However, according to the above definition the loaded coefficient value is the maximum value among the coefficient values of the same row, so that a cross-loading by this definition cannot occur. If you look at the table above, you can confirm that there is no cross-loading by this rule at all. Another variant of this rule is that the loading must be at least .2 or some other arbitrary number higher than any of the potential cross-loadings. According to this rule, for example all the ACSI items would cross-load on both factors regardless whether pattern or structure coefficients are inspected.

The second rule is what we call column comparison (Thompson, 1997). That is, 'for *a factor*, if the absolute value of the coefficient between the loaded item and the factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between any unloaded items and the factor (e.g., PERV1-Factor1), the item is cross-loaded. ' Even with this rule, there is no cross-loading in the above pattern matrix. However, in the structure matrix there are cross-loadings in CUEX1, CUEX2, CUEX3, and PERQ3. For example, the coefficient of CUEX1-Facor1 is .536, of which absolute value is smaller than that of the coefficient of .688 of PERV1-Factor1.

### Problems / Limitations

One disadvantage of this technique is the lack of consensus on exactly what cross-loadings are. More fundamentally, examining cross-loadings to assess discriminant validity does not fit the definition of discriminant validity. For further discussion, see the main text.

## Factor correlation (obtained from EFA)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

As each variable name implies, our example is expected to include four factors. However, as a result of the usual procedure for determining the number of factors, it was determined to be two, which was used for simplicity in the previous example. One possible alternative is to analyze the number of factors fixed at four. The following table shows the results.

# Four factor solution, promax rotation. Factor correlations are printed out  
# by default  
  
fa(ECSICovData, nfactors = 4, rotate = "promax")

## Factor Analysis using method = minres  
## Call: fa(r = ECSICovData, nfactors = 4, rotate = "promax")  
## Standardized loadings (pattern matrix) based upon correlation matrix  
## MR1 MR4 MR3 MR2 h2 u2 com  
## ACSI1 0.76 0.22 0.00 -0.06 0.84 0.156 1.2  
## ACSI2 0.70 0.23 -0.08 -0.04 0.69 0.309 1.3  
## ACSI3 0.55 0.24 0.06 -0.03 0.59 0.412 1.4  
## CUEX1 -0.11 0.02 0.96 -0.03 0.82 0.183 1.0  
## CUEX2 0.15 -0.02 0.53 0.07 0.44 0.562 1.2  
## CUEX3 -0.04 0.04 0.04 0.99 1.00 0.005 1.0  
## PERQ1 0.84 -0.04 0.17 -0.07 0.80 0.205 1.1  
## PERQ2 0.96 -0.10 0.02 -0.07 0.77 0.230 1.0  
## PERQ3 0.63 -0.06 -0.09 0.17 0.38 0.621 1.2  
## PERV1 0.21 0.70 0.02 0.01 0.76 0.240 1.2  
## PERV2 -0.04 0.92 0.00 0.04 0.80 0.203 1.0  
##   
## MR1 MR4 MR3 MR2  
## SS loadings 3.74 1.82 1.31 1.00  
## Proportion Var 0.34 0.17 0.12 0.09  
## Cumulative Var 0.34 0.51 0.62 0.72  
## Proportion Explained 0.47 0.23 0.17 0.13  
## Cumulative Proportion 0.47 0.71 0.87 1.00  
##   
## With factor correlations of   
## MR1 MR4 MR3 MR2  
## MR1 1.00 0.74 0.60 0.36  
## MR4 0.74 1.00 0.45 0.16  
## MR3 0.60 0.45 1.00 0.37  
## MR2 0.36 0.16 0.37 1.00  
##   
## Mean item complexity = 1.1  
## Test of the hypothesis that 4 factors are sufficient.  
##   
## The degrees of freedom for the null model are 55 and the objective function was 7.2  
## The degrees of freedom for the model are 17 and the objective function was 0.04   
##   
## The root mean square of the residuals (RMSR) is 0.01   
## The df corrected root mean square of the residuals is 0.02   
##   
## Fit based upon off diagonal values = 1  
## Measures of factor score adequacy   
## MR1 MR4 MR3 MR2  
## Correlation of (regression) scores with factors 0.97 0.94 0.93 1.00  
## Multiple R square of scores with factors 0.94 0.89 0.87 0.99  
## Minimum correlation of possible factor scores 0.88 0.78 0.73 0.99

### Problems / Limitations

EFA is a technique of 'letting the data speak', and the theoretical background and the intention of the researcher are not taken into consideration at all. Therefore, EFA often produces results that are difficult to interpret. The following shows the derived pattern matrix. Many items are 'loaded' on unintended factors. This discussion shows why CFA, not EFA, should be used when looking for factor correlation.

We will now describe techniques for evaluating discriminant validity using the lavaan package (Rosseel, 2012) for SEM.

## Factor correlation (point estimate)

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

Two methods are used to determine the scale of the latent variable in SEM:

1. Fix one of the path coefficients between the latent variable and the corresponding manifest variables (i.e., factor loadings) to a non-zero value (typically 1), or
2. Fix the variance of latent variables to a non-zero value (typically 1).

The first style is the lavaan default, so it is used more often. However, the second style is more convenient for estimating factor correlations. We use the second style by setting the std.lv option. The advantage of fixing the variance of latent variables to 1 is that the estimated factor covariance are correlations that can be interpreted directly without any further calculation. That is, the correlation table as shown in Table 41 can be obtained from the following results.

Table 41 Factor correlation estimates from R

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .612 | 1.000 |  |  |
| PERQ | .957 | .698 | 1.000 |  |
| PERV | .875 | .526 | f770 | 1.000 |

CFAest <- cfa(CFAmodel, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling  
  
summary(CFAest)

## lavaan 0.6-3 ended normally after 33 iterations  
##   
## Optimization method NLMINB  
## Number of free parameters 28  
##   
## Number of observations 10417  
##   
## Estimator ML  
## Model Fit Test Statistic 1614.286  
## Degrees of freedom 38  
## P-value (Chi-square) 0.000  
##   
## Parameter Estimates:  
##   
## Information Expected  
## Information saturated (h1) model Structured  
## Standard Errors Standard  
##   
## Latent Variables:  
## Estimate Std.Err z-value P(>|z|)  
## ACSI =~   
## ACSI1 1.853 0.015 122.928 0.000  
## ACSI2 1.745 0.017 103.059 0.000  
## ACSI3 1.459 0.016 91.645 0.000  
## CUEX =~   
## CUEX1 1.581 0.021 76.579 0.000  
## CUEX2 1.647 0.022 76.095 0.000  
## CUEX3 1.038 0.024 42.959 0.000  
## PERQ =~   
## PERQ1 1.711 0.015 116.266 0.000  
## PERQ2 1.563 0.014 109.742 0.000  
## PERQ3 0.978 0.016 62.376 0.000  
## PERV =~   
## PERV1 1.643 0.014 115.125 0.000  
## PERV2 1.856 0.018 102.481 0.000  
##   
## Covariances:  
## Estimate Std.Err z-value P(>|z|)  
## ACSI ~~   
## CUEX 0.612 0.009 71.846 0.000  
## PERQ 0.957 0.002 384.934 0.000  
## PERV 0.875 0.004 226.986 0.000  
## CUEX ~~   
## PERQ 0.698 0.008 90.904 0.000  
## PERV 0.526 0.010 54.787 0.000  
## PERQ ~~   
## PERV 0.770 0.006 139.827 0.000  
##   
## Variances:  
## Estimate Std.Err z-value P(>|z|)  
## .ACSI1 0.566 0.013 43.222 0.000  
## .ACSI2 1.365 0.022 62.783 0.000  
## .ACSI3 1.480 0.022 66.232 0.000  
## .CUEX1 1.909 0.044 43.451 0.000  
## .CUEX2 2.128 0.048 44.160 0.000  
## .CUEX3 4.212 0.063 66.969 0.000  
## .PERQ1 0.682 0.015 46.769 0.000  
## .PERQ2 0.796 0.015 54.789 0.000  
## .PERQ3 1.933 0.028 69.570 0.000  
## .PERV1 0.539 0.017 31.211 0.000  
## .PERV2 1.396 0.028 50.375 0.000  
## ACSI 1.000   
## CUEX 1.000   
## PERQ 1.000   
## PERV 1.000

### Problems / Limitations

The point estimate provides only limited information about the parameter. Particularly, a single point estimate does not tell us anything about how certain we are about the estimate. A better alternative is an interval estimate in the form of a 95% confidence interval, which gives information on the maximum and minimum values of the parameter when the assumptions are met at a given confidence level.

## Factor correlation (whether the confidence interval includes 1)

### How often is it used?

AMJ 0%, JAP 0%, ORM 5.0%

### How to obtain

Confidence intervals can be obtained by setting the ci argument of the summary method to true.

Table 42 Confidence intervals for correlations obtained from R

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | [.596,.629] | 1.000 |  |  |
| PERQ | [.952,.962] | [.683,.713] | 1.000 |  |
| PERV | [.867,.882] | [.507,.545] | [.759,.780] | 1.000 |

summary(CFAest, ci = TRUE)

## lavaan 0.6-3 ended normally after 33 iterations  
##   
## Optimization method NLMINB  
## Number of free parameters 28  
##   
## Number of observations 10417  
##   
## Estimator ML  
## Model Fit Test Statistic 1614.286  
## Degrees of freedom 38  
## P-value (Chi-square) 0.000  
##   
## Parameter Estimates:  
##   
## Information Expected  
## Information saturated (h1) model Structured  
## Standard Errors Standard  
##   
## Latent Variables:  
## Estimate Std.Err z-value P(>|z|) ci.lower ci.upper  
## ACSI =~   
## ACSI1 1.853 0.015 122.928 0.000 1.823 1.882  
## ACSI2 1.745 0.017 103.059 0.000 1.712 1.778  
## ACSI3 1.459 0.016 91.645 0.000 1.428 1.491  
## CUEX =~   
## CUEX1 1.581 0.021 76.579 0.000 1.541 1.622  
## CUEX2 1.647 0.022 76.095 0.000 1.604 1.689  
## CUEX3 1.038 0.024 42.959 0.000 0.990 1.085  
## PERQ =~   
## PERQ1 1.711 0.015 116.266 0.000 1.682 1.740  
## PERQ2 1.563 0.014 109.742 0.000 1.535 1.591  
## PERQ3 0.978 0.016 62.376 0.000 0.947 1.009  
## PERV =~   
## PERV1 1.643 0.014 115.125 0.000 1.615 1.671  
## PERV2 1.856 0.018 102.481 0.000 1.820 1.891  
##   
## Covariances:  
## Estimate Std.Err z-value P(>|z|) ci.lower ci.upper  
## ACSI ~~   
## CUEX 0.612 0.009 71.846 0.000 0.596 0.629  
## PERQ 0.957 0.002 384.934 0.000 0.952 0.962  
## PERV 0.875 0.004 226.986 0.000 0.867 0.882  
## CUEX ~~   
## PERQ 0.698 0.008 90.904 0.000 0.683 0.713  
## PERV 0.526 0.010 54.787 0.000 0.507 0.545  
## PERQ ~~   
## PERV 0.770 0.006 139.827 0.000 0.759 0.780  
##   
## Variances:  
## Estimate Std.Err z-value P(>|z|) ci.lower ci.upper  
## .ACSI1 0.566 0.013 43.222 0.000 0.541 0.592  
## .ACSI2 1.365 0.022 62.783 0.000 1.323 1.408  
## .ACSI3 1.480 0.022 66.232 0.000 1.436 1.524  
## .CUEX1 1.909 0.044 43.451 0.000 1.823 1.995  
## .CUEX2 2.128 0.048 44.160 0.000 2.033 2.222  
## .CUEX3 4.212 0.063 66.969 0.000 4.089 4.336  
## .PERQ1 0.682 0.015 46.769 0.000 0.654 0.711  
## .PERQ2 0.796 0.015 54.789 0.000 0.768 0.825  
## .PERQ3 1.933 0.028 69.570 0.000 1.879 1.988  
## .PERV1 0.539 0.017 31.211 0.000 0.505 0.573  
## .PERV2 1.396 0.028 50.375 0.000 1.342 1.450  
## ACSI 1.000 1.000 1.000  
## CUEX 1.000 1.000 1.000  
## PERQ 1.000 1.000 1.000  
## PERV 1.000 1.000 1.000

### Problems / Limitations

There is no problem with this technique itself, but the problem lies in the way we have used this technique this far. When evaluating discriminant validity, we have examined whether the interval estimates of factor correlation include one (i.e., perfect correlation). That is, if the maximum value of the confidence interval is less than 1, it is determined that there is no discriminant validity problem. This is problematic because almost all data will meet these criteria as long as the sample size is large enough. For example, in the above example, the factor correlation between ACSI-PERQ is very high, but its confidence interval does not include 1.

## Techniques using model fit indices: no comparison

### How often is it used?

AMJ 11.1%, JAP 1.4%, ORM 0%

### How to obtain

The values of the model fit indices are automatically calculated in Lavaan. The statistic is reported by default and more fit indices can be obtained by using the fit.indices argument of the summary method. For example, this case the fit indices are = 1614.286, p = 0.000, GFI .973, TLI 970, CFI .979, and RMSEA .063. The value shows that the model does not fit exactly. While there are SEM model evaluation guidelines that provide cutoffs for the other indices and our values would be considered acceptable against these cutoffs, we nevertheless suggest that researchers diagnose their models to understand the source of misfit before declaring misfit acceptable (Kline, 2011, Chapter 8). However, applying this technique makes no indication of any problem in the discriminant validity of these data.

##### Techniques using model fit indices: no comparison #####  
  
summary(CFAest, fit.measures = TRUE)

## lavaan 0.6-3 ended normally after 33 iterations  
##   
## Optimization method NLMINB  
## Number of free parameters 28  
##   
## Number of observations 10417  
##   
## Estimator ML  
## Model Fit Test Statistic 1614.286  
## Degrees of freedom 38  
## P-value (Chi-square) 0.000  
##   
## Model test baseline model:  
##   
## Minimum Function Test Statistic 75026.313  
## Degrees of freedom 55  
## P-value 0.000  
##   
## User model versus baseline model:  
##   
## Comparative Fit Index (CFI) 0.979  
## Tucker-Lewis Index (TLI) 0.970  
##   
## Loglikelihood and Information Criteria:  
##   
## Loglikelihood user model (H0) -204807.453  
## Loglikelihood unrestricted model (H1) -204000.310  
##   
## Number of free parameters 28  
## Akaike (AIC) 409670.906  
## Bayesian (BIC) 409873.940  
## Sample-size adjusted Bayesian (BIC) 409784.960  
##   
## Root Mean Square Error of Approximation:  
##   
## RMSEA 0.063  
## 90 Percent Confidence Interval 0.060 0.066  
## P-value RMSEA <= 0.05 0.000  
##   
## Standardized Root Mean Square Residual:  
##   
## SRMR 0.029  
##   
## Parameter Estimates:  
##   
## Information Expected  
## Information saturated (h1) model Structured  
## Standard Errors Standard  
##   
## Latent Variables:  
## Estimate Std.Err z-value P(>|z|)  
## ACSI =~   
## ACSI1 1.853 0.015 122.928 0.000  
## ACSI2 1.745 0.017 103.059 0.000  
## ACSI3 1.459 0.016 91.645 0.000  
## CUEX =~   
## CUEX1 1.581 0.021 76.579 0.000  
## CUEX2 1.647 0.022 76.095 0.000  
## CUEX3 1.038 0.024 42.959 0.000  
## PERQ =~   
## PERQ1 1.711 0.015 116.266 0.000  
## PERQ2 1.563 0.014 109.742 0.000  
## PERQ3 0.978 0.016 62.376 0.000  
## PERV =~   
## PERV1 1.643 0.014 115.125 0.000  
## PERV2 1.856 0.018 102.481 0.000  
##   
## Covariances:  
## Estimate Std.Err z-value P(>|z|)  
## ACSI ~~   
## CUEX 0.612 0.009 71.846 0.000  
## PERQ 0.957 0.002 384.934 0.000  
## PERV 0.875 0.004 226.986 0.000  
## CUEX ~~   
## PERQ 0.698 0.008 90.904 0.000  
## PERV 0.526 0.010 54.787 0.000  
## PERQ ~~   
## PERV 0.770 0.006 139.827 0.000  
##   
## Variances:  
## Estimate Std.Err z-value P(>|z|)  
## .ACSI1 0.566 0.013 43.222 0.000  
## .ACSI2 1.365 0.022 62.783 0.000  
## .ACSI3 1.480 0.022 66.232 0.000  
## .CUEX1 1.909 0.044 43.451 0.000  
## .CUEX2 2.128 0.048 44.160 0.000  
## .CUEX3 4.212 0.063 66.969 0.000  
## .PERQ1 0.682 0.015 46.769 0.000  
## .PERQ2 0.796 0.015 54.789 0.000  
## .PERQ3 1.933 0.028 69.570 0.000  
## .PERV1 0.539 0.017 31.211 0.000  
## .PERV2 1.396 0.028 50.375 0.000  
## ACSI 1.000   
## CUEX 1.000   
## PERQ 1.000   
## PERV 1.000

### Problems / Limitations

The fit of the proposed model has nothing to do with the discriminant validity. Assessing discriminant validity requires a well-fitting model, but the model fit itself does not inform us about discriminant validity. To assess discriminant validity using model fit indices, a comparison with other alternative models is needed. The question is which alternative model to compare.

## Techniques using model fit indices: compared to nested models with fewer factors

### How often is it used?

AMJ 29.6%, JAP 58.9%, ORM 25.0%

As far as we know, there are no guidelines-type article that recommends the use of this technique for evaluating discriminant validity. Surprisingly, however, this technique is the most commonly used technique in applied psychology.

### How to obtain

Because there is no authoritative source, this technique is being applied in a wide variety of ways. A typical method is as follows. Suppose the proposed model is composed of N factors. Then, we can construct (N - 1) -factor models, (N - 2)-factor models … and a 1-factor model by merging some of the factors into one. This technique compares the fit indices of all these (or some arbitrarily selected) alternative models with the originally proposed models.

In our ACSI example, the factor correlation between ACSI and PERQ is the highest, so you can compare an alternative three-factor model that combines the two factors into one factor with the originally proposed model. (Of course, it is also possible to review comparisons with all possible three-factor, two-factor, and one-factor models.) The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

CFAmodel2 <- "ACSIPERQ =~ ACSI1 + ACSI2 + ACSI3 + PERQ1 + PERQ2 + PERQ3  
CUEX =~ CUEX1 + CUEX2 + CUEX3  
PERV =~ PERV1 + PERV2"  
  
CFAest2 <- cfa(CFAmodel2, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling  
  
lavTestLRT(CFAest, CFAest2)

## Chi Square Difference Test  
##   
## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
## CFAest 38 409671 409874 1614.3   
## CFAest2 41 411023 411204 2972.5 1358.2 3 < 2.2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

### Problems / Limitations

Notice that the difference in degrees of freedom between the proposed model and the alternative model is three. Originally, our interest was a high correlation between ACSI and PERQ, but merging the two factors into one adds additional constraints. In other words, the above model is equivalent to the original model with the following three constraints.

1. The correlation between ACSI and PERQ (.957) is 1.
2. The correlation between ACSI and CUEX (.612) and the correlation between PERQ and CUEX (.698) are equal.
3. The correlation between ACSI and PERV (.875) and the correlation between PERQ and PERV (. 770) are equal.

That is, the following model:

CFAmodel3 <- "ACSI =~ ACSI1 + ACSI2 + ACSI3  
CUEX =~ CUEX1 + CUEX2 + CUEX3  
PERQ =~ PERQ1 + PERQ2 + PERQ3  
PERV =~ PERV1 + PERV2  
  
ACSI ~~ 1\*PERQ  
ACSI ~~ a\*CUEX  
PERQ ~~ a\*CUEX  
ACSI ~~ b\*PERV  
PERQ ~~ b\*PERV"  
  
  
CFAest3 <- cfa(CFAmodel3, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling  
  
lavTestLRT(CFAest, CFAest3)

## Chi Square Difference Test  
##   
## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
## CFAest 38 409671 409874 1614.3   
## CFAest3 41 411023 411204 2972.5 1358.2 3 < 2.2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Of these, only the first constraint is truly relevant to discriminant validity, and a strategy that focuses only on the necessary constraints is needed.

## Techniques using model fit indices: comparison against model with correlation fixed at 1

### How often is it used?

AMJ 14.8%, JAP 1.4%, ORM 10.0%

### How to obtain

For all possible latent variable pairs, the model with the correlation fixed at 1 is compared with the original model. Here, we present only the model with the correlation between ACSI and PERQ is constrained to 1.

CFAmodel4 <- "ACSI =~ ACSI1 + ACSI2 + ACSI3  
CUEX =~ CUEX1 + CUEX2 + CUEX3  
PERQ =~ PERQ1 + PERQ2 + PERQ3  
PERV =~ PERV1 + PERV2  
  
ACSI ~~ 1\*PERQ"  
  
  
CFAest4 <- cfa(CFAmodel4, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling

## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.

lavTestLRT(CFAest, CFAest4)

## Chi Square Difference Test  
##   
## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
## CFAest 38 409671 409874 1614.3   
## CFAest4 39 410054 410250 1999.8 385.49 1 < 2.2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Although the correlation between ACSI and PERQ is very high at .957, the model with a fixed correlation of 1 has poorer fit indices than the original model. This test seque can be automated to produce all tests with a bit of programming. The idea is to first locate the correlations from the parameter table, produce a list of constraings based on these models, and then update the baseline model using each of the constraints.

# All correlation pairs programmatically  
  
Baseline <- CFAest  
  
# Form the constraints  
constraints <- paste(Baseline@ParTable$lhs[27:32], "~~1\*",  
 Baseline@ParTable$rhs[27:32], sep="")  
  
tests <- lapply(constraints,function(constraint){  
 res <- lavTestLRT(Baseline,update(Baseline, add=constraint))  
 rownames(res)[2] <- constraint # Add the model name  
 res  
})

## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.

## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.  
  
## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.  
  
## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.  
  
## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.  
  
## Warning in lav\_object\_post\_check(object): lavaan WARNING: covariance matrix of latent variables  
## is not positive definite;  
## use lavInspect(fit, "cov.lv") to investigate.

do.call(rbind,c(tests[1],  
 # Remove the baseline from all but first  
 lapply(tests[-1], function(x){x[-1,]})))

## Chi Square Difference Test  
##   
## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)   
## Baseline 38 409671 409874 1614.3   
## ACSI~~1\*CUEX 39 412675 412871 4620.9 3006.6 1 < 2.2e-16 \*\*\*  
## ACSI~~1\*PERQ 39 410054 410250 1999.8 385.5 1 < 2.2e-16 \*\*\*  
## ACSI~~1\*PERV 39 411397 411593 3342.4 1728.1 1 < 2.2e-16 \*\*\*  
## CUEX~~1\*PERQ 39 411853 412049 3798.5 2184.2 1 < 2.2e-16 \*\*\*  
## CUEX~~1\*PERV 39 413313 413509 5258.5 3644.2 1 < 2.2e-16 \*\*\*  
## PERQ~~1\*PERV 39 413359 413555 5304.9 3690.6 1 < 2.2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

### Problems / Limitations

There are no logical flaws in this technique. However, there is a practical problem that almost all data pass the criteria as long as the sample size is large enough because correlations are rarely exactly 1. Thus, applied researchers have preferred a technique that require more difficult-to-pass criteria.

## AVE: compared with the square of factor correlation

### How often is it used?

AMJ 7.4%, JAP 5.5%, ORM 5.0%

Although this technique is not used very often among organizational researchers, it is a standard technique for evaluating discriminant validity in many other business disciplines, such as marketing.

### How to obtain

The reliability function of semTools package (Jorgensen et al., 2018) can be used to calculate average variance extracted. We will now explain how the AVE values are used in the The Fornell-Larcker criterion for assessing discriminant validity. The table below shows the AVE values, the previously reported factor correlations on the lower-triangle and the squares of the factor correlations are shown on the upper triangle (e.g.,.

##### AVE: compared with the square of factor correlation #####  
  
AVEs <- reliability(CFAest)["avevar",1:4]  
  
comparisonMatrix <- inspect(CFAest, what = "cor.lv")  
class(comparisonMatrix) <- "matrix"  
  
diag(comparisonMatrix) <- AVEs  
  
comparisonMatrix[upper.tri(comparisonMatrix)] <-   
 comparisonMatrix[upper.tri(comparisonMatrix)]^2  
  
comparisonMatrix

## ACSI CUEX PERQ PERV  
## ACSI 0.7161246 0.3751215 0.9156972 0.7649339  
## CUEX 0.6124717 0.4325880 0.4877437 0.2766892  
## PERQ 0.9569207 0.6983865 0.6496510 0.5922257  
## PERV 0.8746050 0.5260125 0.7695620 0.7605259

The Fornell-Larcker criterion for assessing discriminant validity is that for every pair of latent variables, the square of the factor correlation must be less than the AVE values of both latent variables. The acronym AVE/SV comes from the fact that the square of the factor correlation is also called shared variance. The above example fails this criterion for three pairs of latent variables: 1) The shared variance between ACSI and PERQ is 0.9156972, which is larger than ACSI's AVE value of 0.7161246 and PERQ's AVE value of 0.6496510. 2) The shared variance between ACSI and PERV is 0.7649339, which is larger than the ACSI’s AVE value of 0.7161246. 3) The shared variance between CUEX and PERQ is 0.4877437, which is greater than the CUEX’s AVE value of 0.4325880.

### Problems / Limitations

Few methodological studies have seriously considered this technique, and the logical problems of this technique are almost unknown. Essentially, AVE is a weighted average of item-level reliabilities. While factor correlation is related to discriminant validity, as a comparison point, AVE has little relevance. Moreover, given methodological literature’s lack of attention to this technique, it is often is misapplied in a variety of ways. The most common misuse is that AVE is compared to a value other than the square of the factor correlation.

## AVE: compared with the square of scale score correlation

### How often is it used?

AMJ 3.7%, JAP 1.4%, ORM 0%

Many users are more familiar with the process of obtaining scale score correlations than factor correlations. Therefore, unlike the original proposal of Fornell and Larcker (1981a), studies comparing AVE to the square of the scale score correlation are often found.

### How to obtain

AVE values are shown on the diagonals (in italics). The previously reported scale score correlation (Table 2) is shown in the sub-diagonals. The square of the scale score correlation is displayed in the super-diagonals (e.g0.2360396 .

comparisonMatrix <- scaleScoreCorrelations  
  
diag(comparisonMatrix) <- AVEs  
  
comparisonMatrix[upper.tri(comparisonMatrix)] <-   
 comparisonMatrix[upper.tri(comparisonMatrix)]^2  
  
comparisonMatrix

## ACSI CUEX PERQ PERV  
## ACSI 0.7161246 0.2360396 0.6676081 0.5844350  
## CUEX 0.4858390 0.4325880 0.3021886 0.1633090  
## PERQ 0.8170729 0.5497168 0.6496510 0.4154539  
## PERV 0.7644835 0.4041151 0.6445571 0.7605259

The Fornell-Larcker criterion was violated in only one case. The square of scale score correlation between ACSI and PERQ is 0.6676081, which is larger than PERQ's AVE value of 0.6496510.

### Problems / Limitations

It is clear that this technique is misuse, so much explanation is not necessary.

## Techniques using model fit indices: comparison against model with correlation fixed at cutoff point less than 1

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

The fact that a correlation is not exactly 1 may be a necessary condition for discriminant validity, but it is difficult to consider it a sufficient condition. In other words, a more realistic test would test against a high but not necessarily perfect correlation. This is the idea in using references values below 1. Which cutoff to use is a matter of subjective judgment, but we can consider candidates such as .85, .9, or .95. This test makes sense only if the correlation estimate is below the the cutoff; if it does not, the discriminant validity test should be considered as failed. To demonstrate this test, we compare the correlation between ACSI and PERV against a fixed cutoff of .96, which in this case is just an arbitrarily chosen cutoff that is greater than the estimated correlation of .957.

CFAmodel5 <- "ACSI =~ ACSI1 + ACSI2 + ACSI3  
CUEX =~ CUEX1 + CUEX2 + CUEX3  
PERQ =~ PERQ1 + PERQ2 + PERQ3  
PERV =~ PERV1 + PERV2  
  
ACSI ~~ .96\*PERQ"  
  
  
CFAest5 <- cfa(CFAmodel5, sample.cov = ECSICovData, sample.nobs = N,  
 std.lv = TRUE) # use alternative scaling  
  
lavTestLRT(CFAest, CFAest5)

## Chi Square Difference Test  
##   
## Df AIC BIC Chisq Chisq diff Df diff Pr(>Chisq)  
## CFAest 38 409671 409874 1614.3   
## CFAest5 39 409671 409866 1615.9 1.6092 1 0.2046

If the reference point is .96, the correlation between ACSI and PERV is not significantly less than that, so we can assess that there is a problem in the discriminant validity.

### Problems / Limitations

This technique has the advantage of being more flexible than the previous techniques, but there are also limitations. There is room for criticism that the user must determine the reference value, which is an arbitrary choice.

## Structure coefficients (obtained from CFA)

### How often is it used?

AMJ 7.4%, JAP 0%, ORM 0%

### How to obtain

A typical CFA is an independent clusters (IC) model, where each item loads on just one factor and the loading value with other factors is zero. In the case of a CFA model specification, the term loading invariably refers to the factor pattern coefficients because the factor structure coefficients are not model parameters that are estimated, but are something that can be calculated post-estimation.

Becauese structure coefficients are simply correlations, they can be obtained from the full implied correlation matrix containing both latent and observed variables using the inspect function.

inspect(CFAest, what = "cor.all")[1:11,12:15]

## ACSI CUEX PERQ PERV  
## ACSI1 0.9264803 0.5674429 0.8865682 0.8103044  
## ACSI2 0.8308958 0.5089001 0.7951014 0.7267056  
## ACSI3 0.7680865 0.4704312 0.7349979 0.6717723  
## CUEX1 0.4611966 0.7530089 0.5258913 0.3960921  
## CUEX2 0.4584693 0.7485560 0.5227814 0.3937498  
## CUEX3 0.2763839 0.4512599 0.3151539 0.2373684  
## PERQ1 0.8617431 0.6289234 0.9005377 0.6930196  
## PERQ2 0.8310323 0.6065098 0.8684443 0.6683217  
## PERQ3 0.5504902 0.4017626 0.5752726 0.4427079  
## PERV1 0.7985741 0.4802854 0.7026627 0.9130683  
## PERV2 0.7377571 0.4437082 0.6491500 0.8435317

## CICFA(.9): the proposed technique

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

There are no studies that formally proposed this technique.

### How to obtain

The method of obtaining the upper limit of factor correlation in R has been described above (i.e., Table 42). The difference between the existing technique and CICFA (.9) is what this upper limit is compared to. The existing technique evaluates that there is a problem with discriminant validity if the upper limit is larger than 1. Our concern is not whether the correlation is exactly 1, but whether it is a sufficiently low number of 1, so the existing technique does not fit well with the definition of discriminant validity. We suggest using a number less than 1 as a cutoff. CICFA (.9) evaluates that there is a problem with discriminant validity if the upper limit is larger than .9. The upper limit of the correlation between ACSI-PERQ is .962, which does not pass this criterion.

### Problems / Limitations

The choice of a cutoff is inevitable for a dichotomous (i.e., yes or no) decision, but there is room for criticism that any cutoff is arbitrary. More research is needed as to which cutoff is best, and different cutoffs may be considered depending on the purpose of the study (e.g., CICFA(.85)).

All techniques using Stata

The Stata tutorial is presented as formatted Stata log, which shows the full Stata syntax and output. All lines staring with period (.) are Stata commands and other lines are output. The Stata do file can be downloaded at <https://github.com/eunscho/DiscriminantValidityTutorial> .

## Stata syntax for setting up the data

Set up a dataset based on the covariance matrix. Because Stata has limited support for analysis of covariance data, we will generate a dataset with matching covariances

-------------------------------------------------------------------------------------

. matrix S = (4.00, ///

3.23, 4.41, ///

2.66, 2.67, 3.61, ///

1.81, 1.50, 1.56, 4.41, ///

1.85, 1.62, 1.63, 2.63, 4.84, ///

1.24, 1.16, 1.09, 1.55, 1.72, 5.29, ///

3.04, 2.83, 2.35, 2.07, 1.96, 1.31, 3.61, ///

2.81, 2.57, 2.19, 1.55, 1.74, 1.12, 2.67, 3.24, ///

1.73, 1.64, 1.32, 0.89, 1.05, 1.41, 1.62, 1.62, 2.89, ///

2.66, 2.49, 2.12, 1.40, 1.43, 0.95, 2.22, 2.01, 1.25, 3.24, ///

2.99, 2.86, 2.42, 1.52, 1.50, 1.01, 2.34, 2.14, 1.31, 3.05, 4.84)

. corr2data acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2, ///

n(10417) cov(S) cstorage(lower) clear

(obs 10,417)

## Scale score correlation

### How often is it used?

AMJ 25.9%, JAP 11.0%, ORM 55.0%

This is probably the most easily understood among the various techniques, so it is commonly used.

### How to obtain

Scales scores are typically the unweighted sum or mean of observed item scores.

. gen acsi = acsi1 + acsi2 + acsi3

. gen cuex = cuex1 + cuex2 + cuex3

. gen perq = perq1 + perq2 + perq3

. gen perv = perv1 + perv2

. correlate acsi cuex perq perv

(obs=10,417)

| acsi cuex perq perv

-------------+------------------------------------

acsi | 1.0000

cuex | 0.4858 1.0000

perq | 0.8171 0.5497 1.0000

perv | 0.7645 0.4041 0.6446 1.0000

### Problems / Limitations

Scale score correlation is a function of 'true correlation' and scale score reliability. That is, a low scale score correlation may be due to a low true correlation, or a low reliability. Therefore, these correlations alone do not allow any conclusions about discriminant validity to be drawn.

## Disattenuated correlation using tau-equivalent reliability

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

As mentioned above, scale score correlation is a function of 'true correlation' and scale score reliability. Therefore, the true correlation can be estimated by adjusting the scale score correlation to account for its unreliability. The estimate of true correlation obtained in this way is called disattenuated correlation. The effectiveness of this technique depends on a) how accurate the reliability estimates are used and b) to the extent that the indicators are contaminated by non-random measurement error. In typical applications, non-random measurement error is assumed to not exist.

There are many ways to estimate scale score reliability, but the most commonly used is tau-equivalent reliability. This reliability coefficient is often incorrectly known as Cronbach's alpha. Kuder and Richardson (1937), not Cronbach (1951), developed this coefficient first (see Cho & Kim, 2015). Tau-equivalent reliability can be calculated using the alpha command. After the reliability estimates have been calculated, disattenuated correlationcan be calculated as follows:

We apply first the disattenuation formula to one correlation in a single equation and then apply matrix algebra tools to disattenuate all correlations simultaneously and print out the full disattenuated correlation matrix.

. alpha acsi1 acsi2 acsi3

Test scale = mean(unstandardized items)

Average interitem covariance: 2.853333

Number of items in the scale: 3

Scale reliability coefficient: 0.8813

. scalar rel1 = r(alpha)

. alpha perq1 perq2 perq3

Test scale = mean(unstandardized items)

Average interitem covariance: 1.97

Number of items in the scale: 3

Scale reliability coefficient: 0.8224

. scalar rel3 = r(alpha)

. correlate acsi perq

(obs=10,417)

| acsi perq

-------------+------------------

acsi | 1.0000

perq | 0.8171 1.0000

. display r(rho)/sqrt(rel1\*rel3)

.9597941

. quietly alpha cuex1 cuex2 cuex3

. scalar rel2 = r(alpha)

. quietly alpha perv1 perv2

. scalar rel4 = r(alpha)

. matrix allrel = (rel1,rel2,rel3,rel4)

. quietly correlate acsi cuex perq perv

. mata:

----------------------------- mata (type end to exit) -------------------------------

: rel = sqrt(cross(st\_matrix("allrel"),st\_matrix("allrel")))

: \_diag(rel, 1)

: st\_matrix("r(C)"):/rel

[symmetric]

1 2 3 4

+---------------------------------------------------------+

1 | 1 |

2 | .6313365276 1 |

3 | .9597940973 .7394865758 1 |

4 | .8779576943 .5314770133 .7662837795 1 |

+---------------------------------------------------------+

: end

-------------------------------------------------------------------------------------

### Problems / Limitations

Tau-equivalent reliability is widely misunderstood and misused (Cho, 2016; Cho & Kim, 2015). Perhaps the most common misconception is that it is a general reliability coefficient that can be applied to all data. Tau-equivalent reliability is based on the assumption that each variable is influenced by the true score to the same extent, which implies equal covariances as shown in Table 5. For example, to satisfy tau-equivalence, the covariances between ACSI1, ACSI2, and ACSI3 that make up ACSI must all have the same value. The covariance of ACSI1-ACSI3 is 2.66 and the covariance of ACSI2-ACSI3 is 2.67, so it appears that tau-equivalence is met. However, the covariance of ACSI1-ACSI2 is 3.23, which differs greatly from the other two values and thus does not meet tau-equivalence. Therefore, a technique that does not depend on the assumption of tau-equivalence is needed.

Table 43 Examples of parallel, tau-equivalent, and congeneric covariances

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A. Parallel data | | | | B. Tau-equivalent data | | | | C. Congeneric data | | | |
|  | X1 | X2 | X3 |  | X1 | X2 | X3 |  | X1 | X2 | X3 |
| X1 | 10 | 4 | 4 | X1 | 10 | 4 | 4 | X1 | 10 | 2 | 3 |
| X2 | 4 | 10 | 4 | X2 | 4 | 9 | 4 | X2 | 2 | 9 | 6 |
| X3 | 4 | 4 | 10 | X3 | 4 | 4 | 11 | X3 | 3 | 6 | 11 |
| All three values are correct | | | | and are correct | | | | Only is correct  , | | | |

## Disattenuated correlation using parallel reliability (so called HTMT)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

This technique is rarely used by organizational researchers yet. However, its popularity has exploded in other related fields after Henseler, Ringle and Sarstedt (2015) has introduced this technique under the name Heterotrait-Monotrait (HTMT).

### How to obtain

Parallel reliability is often mistakenly referred to as standardized alpha, which gives a misleading impression that it is a variant of tau-equivalent reliability (i.e., alpha). The two reliability coefficients are based on different assumptions and are independent formulas. As with tau-equivalent reliability, parallel reliability can be calculated using the alpha command. After the reliability estimates are calculated, they are applied to disattenuate unit weighted composites (i.e. sums of standardized items).

. center acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2, ///

standardize prefix(z\_)

(generated variables: z\_acsi1 z\_acsi2 z\_acsi3 z\_cuex1 z\_cuex2 z\_cuex3 z\_perq1 z\_perq2 z\_perq3 z\_perv1 z\_perv2)

. gen z\_acsi = z\_acsi1 + z\_acsi2 + z\_acsi3

. gen z\_cuex = z\_cuex1 + z\_cuex2 + z\_cuex3

. gen z\_perq = z\_perq1 + z\_perq2 + z\_perq3

. gen z\_perv = z\_perv1 + z\_perv2

. alpha acsi1 acsi2 acsi3, std

Test scale = mean(standardized items)

Average interitem correlation: 0.7127

Number of items in the scale: 3

Scale reliability coefficient: 0.8816

. scalar rel1 = r(alpha)

. alpha perq1 perq2 perq3, std

Test scale = mean(standardized items)

Average interitem correlation: 0.6039

Number of items in the scale: 3

Scale reliability coefficient: 0.8206

. scalar rel3 = r(alpha)

. correlate z\_acsi z\_perq

(obs=10,417)

| z\_acsi z\_perq

-------------+------------------

z\_acsi | 1.0000

z\_perq | 0.8121 1.0000

. display r(rho)/sqrt(rel1\*rel3)

.95484489

. quietly alpha cuex1 cuex2 cuex3, std

. scalar rel2 = r(alpha)

. quietly alpha perv1 perv2, std

. scalar rel4 = r(alpha)

. matrix allrel = (rel1,rel2,rel3,rel4)

. quietly correlate z\_acsi z\_cuex z\_perq z\_perv

. mata:

----------------------------- mata (type end to exit) -------------------------------

: rel = sqrt(cross(st\_matrix("allrel"),st\_matrix("allrel")))

: \_diag(rel, 1)

: st\_matrix("r(C)"):/rel

[symmetric]

1 2 3 4

+---------------------------------------------------------+

1 | 1 |

2 | .6335124871 1 |

3 | .9548448852 .7364370085 1 |

4 | .8761793327 .5336834447 .7645405248 1 |

+---------------------------------------------------------+

: end

-------------------------------------------------------------------------------------

### Problems / Limitations

Parallel reliability is less accurate than tau-equivalent reliability. Therefore, disattenuated correlation using parallel reliability is less effective than disattenuated correlation using tau-equivalent reliability for assessing discriminant validity. In addition to tau-equivalence, parallel reliability also requires the assumption that the variance of each item is identical (i.e., the condition of being parallel). In other words, it is more difficult to satisfy the assumption of parallel reliability than to satisfy that of tau-equivalent reliability. Parallel reliability has no computational advantage over tau-equivalent reliability. Therefore, the use of disattenuated correlation using parallel reliability (i.e., HTMT) for the assessment of discriminant validity is difficult to justify.

## Disattenuated correlation using congeneric reliability

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

Congeneric reliability is more accurate than tau-equivalent reliability but less frequently used. This coefficient is often referred to as composite reliability or McDonald's omega. Jöreskog (1971) developed this coefficient first and named its measurement model a congeneric model. The likely reason for the lower adoption of this coefficient compared to the previously presented reliability coefficients is that this coefficient cannot be calculated directly from the data, but requires the estimation of a factor analysis model, typically a CFA.

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) // Alternative scaling

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -204840.68

Iteration 1: log likelihood = -204807.78

Iteration 2: log likelihood = -204807.45

Iteration 3: log likelihood = -204807.45

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -204807.45

( 1) [/]var(ACSI) = 1

( 2) [/]var(CUEX) = 1

( 3) [/]var(PERQ) = 1

( 4) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.852872 .015054 123.08 0.000 1.823366 1.882377

\_cons | -3.99e-09 .0195947 -0.00 1.000 -.0384048 .0384048

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.744797 .0169444 102.97 0.000 1.711587 1.778008

\_cons | -1.11e-08 .0205744 -0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.459294 .0159323 91.59 0.000 1.428067 1.490521

\_cons | -6.56e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.581242 .0205887 76.80 0.000 1.540888 1.621595

\_cons | 2.10e-10 .0205744 0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.646747 .0216043 76.22 0.000 1.604403 1.68909

\_cons | -2.57e-08 .0215541 -0.00 1.000 -.0422453 .0422453

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.037848 .0242034 42.88 0.000 .9904098 1.085285

\_cons | -3.46e-08 .0225339 -0.00 1.000 -.0441656 .0441655

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.710939 .0147078 116.33 0.000 1.682113 1.739766

\_cons | -1.03e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.563125 .0142597 109.62 0.000 1.535176 1.591073

\_cons | 2.50e-09 .0176352 0.00 1.000 -.0345643 .0345644

------------+----------------------------------------------------------------

perq3 |

PERQ | .9779162 .0156875 62.34 0.000 .9471692 1.008663

\_cons | -6.33e-09 .0166555 -0.00 1.000 -.0326441 .0326441

------------+----------------------------------------------------------------

perv1 |

PERV | 1.643444 .0142946 114.97 0.000 1.615427 1.671461

\_cons | -1.46e-08 .0176352 -0.00 1.000 -.0345644 .0345643

------------+----------------------------------------------------------------

perv2 |

PERV | 1.855681 .018127 102.37 0.000 1.820152 1.891209

\_cons | -2.47e-08 .0215541 -0.00 1.000 -.0422453 .0422453

--------------+----------------------------------------------------------------

var(e.acsi1)| .5664824 .012807 .5419292 .592148

var(e.acsi2)| 1.365259 .0218811 1.323039 1.408826

var(e.acsi3)| 1.480114 .0224018 1.436853 1.524679

var(e.cuex1)| 1.909252 .0436581 1.825573 1.996767

var(e.cuex2)| 2.127761 .0480061 2.035721 2.223962

var(e.cuex3)| 4.212365 .0629737 4.090729 4.337617

var(e.perq1)| .6823401 .0144948 .6545141 .711349

var(e.perq2)| .7963304 .0146877 .7680571 .8256445

var(e.perq3)| 1.933403 .0278122 1.879653 1.988689

var(e.perv1)| .5387805 .0174338 .5056719 .5740568

var(e.perv2)| 1.395985 .0278857 1.342386 1.451724

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .6124711 .0085465 71.66 0.000 .5957203 .629222

cov(ACSI,PERQ)| .9569207 .0024519 390.28 0.000 .9521151 .9617263

cov(ACSI,PERV)| .8746049 .0038616 226.49 0.000 .8670364 .8821734

cov(CUEX,PERQ)| .6983861 .0077252 90.40 0.000 .683245 .7135273

cov(CUEX,PERV)| .5260119 .0095995 54.80 0.000 .5071973 .5448265

cov(PERQ,PERV)| .7695618 .0054534 141.12 0.000 .7588734 .7802502

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(38) = 1614.29, Prob > chi2 = 0.0000

We will next use estat framework to get the model parameters as matrices, from which the correct parameter estimates are easier to locate.

. estat framework

Endogenous variables on endogenous variables

| observed

Beta | acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1

-------------+----------------------------------------------------------------------------

observed |

acsi1 | 0

acsi2 | 0 0

acsi3 | 0 0 0

cuex1 | 0 0 0 0

cuex2 | 0 0 0 0 0

cuex3 | 0 0 0 0 0 0

perq1 | 0 0 0 0 0 0 0

perq2 | 0 0 0 0 0 0 0

perq3 | 0 0 0 0 0 0 0

perv1 | 0 0 0 0 0 0 0

perv2 | 0 0 0 0 0 0 0

------------------------------------------------------------------------------------------

| observed

Beta | perq2 perq3 perv1 perv2

-------------+--------------------------------------------

observed |

perq2 | 0

perq3 | 0 0

perv1 | 0 0 0

perv2 | 0 0 0 0

----------------------------------------------------------

Exogenous variables on endogenous variables

| latent

Gamma | ACSI CUEX PERQ PERV

-------------+--------------------------------------------

observed |

acsi1 | 1.852872 0 0 0

acsi2 | 1.744797 0 0 0

acsi3 | 1.459294 0 0 0

cuex1 | 0 1.581242 0 0

cuex2 | 0 1.646747 0 0

cuex3 | 0 1.037848 0 0

perq1 | 0 0 1.710939 0

perq2 | 0 0 1.563125 0

perq3 | 0 0 .9779162 0

perv1 | 0 0 0 1.643444

perv2 | 0 0 0 1.855681

----------------------------------------------------------

Covariances of error variables

| observed

Psi | e.acsi1 e.acsi2 e.acsi3 e.cuex1 e.cuex2 e.cuex3 e.perq1

-------------+----------------------------------------------------------------------------

observed |

e.acsi1 | .5664824

e.acsi2 | 0 1.365259

e.acsi3 | 0 0 1.480114

e.cuex1 | 0 0 0 1.909252

e.cuex2 | 0 0 0 0 2.127761

e.cuex3 | 0 0 0 0 0 4.212365

e.perq1 | 0 0 0 0 0 0 .6823401

e.perq2 | 0 0 0 0 0 0 0

e.perq3 | 0 0 0 0 0 0 0

e.perv1 | 0 0 0 0 0 0 0

e.perv2 | 0 0 0 0 0 0 0

------------------------------------------------------------------------------------------

| observed

Psi | e.perq2 e.perq3 e.perv1 e.perv2

-------------+--------------------------------------------

observed |

e.perq2 | .7963304

e.perq3 | 0 1.933403

e.perv1 | 0 0 .5387805

e.perv2 | 0 0 0 1.395985

----------------------------------------------------------

Intercepts of endogenous variables

| observed

alpha | acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1

-------------+----------------------------------------------------------------------------

\_cons | -3.99e-09 -1.11e-08 -6.56e-09 2.10e-10 -2.57e-08 -3.46e-08 -1.03e-09

------------------------------------------------------------------------------------------

| observed

alpha | perq2 perq3 perv1 perv2

-------------+--------------------------------------------

\_cons | 2.50e-09 -6.33e-09 -1.46e-08 -2.47e-08

----------------------------------------------------------

Covariances of exogenous variables

| latent

Phi | ACSI CUEX PERQ PERV

-------------+--------------------------------------------

latent |

ACSI | 1

CUEX | .6124711 1

PERQ | .9569207 .6983861 1

PERV | .8746049 .5260119 .7695618 1

----------------------------------------------------------

Means of exogenous variables

| latent

kappa | ACSI CUEX PERQ PERV

-------------+--------------------------------------------

mean | 0 0 0 0

----------------------------------------------------------

. matrix loadings = r(Gamma)

. matrix errors = r(Psi)

. matrix factorCorrelations = r(Phi)

After standardized factor loadings () and error variances () have been estimated, congeneric reliability can be calculated using the following formula:

. scalar rel1 = ((loadings[1,1]+loadings[2,1]+loadings[3,1])^2)/ ///

((loadings[1,1]+loadings[2,1]+loadings[3,1])^2 + ///

errors[1,1] + errors[2,2] + errors[3,3])

. scalar rel3 = ((loadings[7,3]+loadings[8,3]+loadings[9,3])^2)/ ///

((loadings[7,3]+loadings[8,3]+loadings[9,3])^2 + ///

errors[7,7] + errors[8,8] + errors[9,9])

. correlate acsi perq

(obs=10,417)

| acsi perq

-------------+------------------

acsi | 1.0000

perq | 0.8171 1.0000

. display r(rho)/sqrt(rel1\*rel3)

.94841162

. scalar rel2 = ((loadings[4,3]+loadings[5,3]+loadings[6,3])^2)/ ///

((loadings[4,3]+loadings[5,3]+loadings[6,3])^2 + ///

errors[4,4] + errors[5,5] + errors[6,6])

. scalar rel4 = ((loadings[10,4]+loadings[11,4])^2)/ ///

((loadings[10,4]+loadings[11,4])^2 + ///

errors[10,10] + errors[11,11])

. matrix allrel = (rel1,rel2,rel3,rel4)

. quietly correlate acsi cuex perq perv

. mata:

----------------------------- mata (type end to exit) -------------------------------

: rel = sqrt(cross(st\_matrix("allrel"),st\_matrix("allrel")))

: \_diag(rel, 1)

: st\_matrix("r(C)"):/rel

[symmetric]

1 2 3 4

+---------------------------------------------------------+

1 | 1 |

2 | . 1 |

3 | .9484116169 . 1 |

4 | .8758320604 . .7562414706 1 |

+---------------------------------------------------------+

: end

-------------------------------------------------------------------------------------

### Problems / Limitations

Disattenuated correlation using congeneric reliability is more general than the previous methods. Generally, the disattenuatio techniques are commonly recommended as alternatives for researchers who have difficulty using SEM software. However, disattenuated correlation technique itself has problems. First, a correlation coefficient that is greater than 1 or less than -1 may be derived. Second, the correlation estimates can be less precise and the calculation process has more steps than when directly estimating factor correlations based on CFA or SEM thus leaving more room for errors. Third, although it is possible to obtain the confidence interval of disattenuated correlation, doing this correctly using equiations is complicated. Another alternative is to apply boostrapping, but this often requires a bit of programming of the statistical software and can be too computationally intensive to be a practical alternative.

## Cross-loadings (obtained from exploratory factor analysis)

### How often is it used?

AMJ 3.7%, JAP 0%, ORM 15.0%

### How to obtain

The inspection of cross-loadings is sometimes suggested as a way for assessing discrimination validity. However, the term cross-loading has at least two different meanings in the literature that need to be explicitly explained.

1. What is a loading? That is, is it a pattern coefficient or a structure coefficient?
2. What is a *cross*-loading? That is, does the determination of the existence of a problematic cross-loadings require absolute comparisons (e.g., cutoff point) or relative comparisons (e.g., other coefficient value)?

The proper application of an exploratory factor analysis (EFA) is a complex subject in itself, so we provide just a brief demonstration instead of fully explaining the analysis. We use the default principla axis factoring, Varimax as the orthogonal rotation, and Promax as the oblique rotation. The number of factors was determined to be two when determined according to the commonly used criterion that the eigenvalues of the factors should all be greater than one[[3]](#footnote-3).

First, let us consider the first question. Factor loadings represent pattern coefficients or structure coefficients depending on the context. The pattern coefficients indicate how the item value changes when the factor’s (unobserved) value changes by one unit holding other factors constant, which is similar to the coefficient in the regression analysis. The structure coefficient is the correlation between items and factors.

Exploratory factor analysis results will generally need to be rotated to make them more interpretable. We start by considering the orthogonal rotation. The word orthogonal is a geometric term, and its corresponding statistical term is ‘being uncorrelated with each other’. That is, the correlation between the two factors is fixed as follows.

Table 44 Factor correlation matrix after orthogonal rotation

|  | Factor 1 | Factor 2 |
| --- | --- | --- |
| Factor 1 | 1 | 0 |
| Factor 2 | 0 | 1 |

The factor structure matrix is the matrix product of the factor pattern matrix and the factor correlation matrix. Because the correlation matrix is an identity matrix (i.e., a matrix composed of diagonal elements of 1 and non-diagonal elements of 0) in orthogonal rotation, the structure matrix and the pattern matrix are identical. The following is the result of Varimax rotation of the example

. factor acsi1-perv2, factors(2)

(obs=10,417)

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 2

Rotation: (unrotated) Number of params = 21

--------------------------------------------------------------------------

Factor | Eigenvalue Difference Proportion Cumulative

-------------+------------------------------------------------------------

Factor1 | 5.70778 5.06981 0.9248 0.9248

Factor2 | 0.63797 0.34400 0.1034 1.0282

Factor3 | 0.29397 0.13791 0.0476 1.0758

Factor4 | 0.15606 0.13988 0.0253 1.1011

Factor5 | 0.01618 0.05895 0.0026 1.1038

Factor6 | -0.04278 0.01276 -0.0069 1.0968

Factor7 | -0.05554 0.02434 -0.0090 1.0878

Factor8 | -0.07988 0.03603 -0.0129 1.0749

Factor9 | -0.11590 0.05366 -0.0188 1.0561

Factor10 | -0.16956 0.00712 -0.0275 1.0286

Factor11 | -0.17668 . -0.0286 1.0000

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Factor loadings (pattern matrix) and unique variances

-------------------------------------------------

Variable | Factor1 Factor2 | Uniqueness

-------------+--------------------+--------------

acsi1 | 0.9025 -0.1251 | 0.1699

acsi2 | 0.8092 -0.1716 | 0.3157

acsi3 | 0.7651 -0.0687 | 0.4099

cuex1 | 0.5401 0.4227 | 0.5296

cuex2 | 0.5348 0.4233 | 0.5348

cuex3 | 0.3641 0.3118 | 0.7702

perq1 | 0.8659 0.0744 | 0.2446

perq2 | 0.8323 0.0064 | 0.3072

perq3 | 0.5643 0.0759 | 0.6758

perv1 | 0.8024 -0.2272 | 0.3046

perv2 | 0.7335 -0.2647 | 0.3920

-------------------------------------------------

. rotate, orthogonal varimax

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 2

Rotation: orthogonal varimax (Kaiser off) Number of params = 21

--------------------------------------------------------------------------

Factor | Variance Difference Proportion Cumulative

-------------+------------------------------------------------------------

Factor1 | 4.84070 3.33564 0.7843 0.7843

Factor2 | 1.50505 . 0.2439 1.0282

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Rotated factor loadings (pattern matrix) and unique variances

-------------------------------------------------

Variable | Factor1 Factor2 | Uniqueness

-------------+--------------------+--------------

acsi1 | 0.8734 0.2593 | 0.1699

acsi2 | 0.8077 0.1784 | 0.3157

acsi3 | 0.7250 0.2539 | 0.4099

cuex1 | 0.3169 0.6083 | 0.5296

cuex2 | 0.3118 0.6066 | 0.5348

cuex3 | 0.2026 0.4345 | 0.7702

perq1 | 0.7576 0.4259 | 0.2446

perq2 | 0.7551 0.3500 | 0.3072

perq3 | 0.4824 0.3024 | 0.6758

perv1 | 0.8245 0.1250 | 0.3046

perv2 | 0.7773 0.0623 | 0.3920

-------------------------------------------------

Factor rotation matrix

--------------------------------

| Factor1 Factor2

-------------+------------------

Factor1 | 0.9105 0.4136

Factor2 | -0.4136 0.9105

--------------------------------

. estat structure

Structure matrix: correlations between variables and varimax rotated common factors

----------------------------------

Variable | Factor1 Factor2

-------------+--------------------

acsi1 | 0.8734 0.2593

acsi2 | 0.8077 0.1784

acsi3 | 0.7250 0.2539

cuex1 | 0.3169 0.6083

cuex2 | 0.3118 0.6066

cuex3 | 0.2026 0.4345

perq1 | 0.7576 0.4259

perq2 | 0.7551 0.3500

perq3 | 0.4824 0.3024

perv1 | 0.8245 0.1250

perv2 | 0.7773 0.0623

----------------------------------

While this example would suggest that there is not much difference between the pattern and structure matrices, it would be a mistake to assume so. Orthogonal rotation is based on an unrealistic assumption that factors are uncorrelated and should thus be avoided in research that aims to study correlations between constructs. The following is the result of Promax rotation of the example.

. factor acsi1-perv2, factors(2)

(obs=10,417)

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 2

Rotation: (unrotated) Number of params = 21

--------------------------------------------------------------------------

Factor | Eigenvalue Difference Proportion Cumulative

-------------+------------------------------------------------------------

Factor1 | 5.70778 5.06981 0.9248 0.9248

Factor2 | 0.63797 0.34400 0.1034 1.0282

Factor3 | 0.29397 0.13791 0.0476 1.0758

Factor4 | 0.15606 0.13988 0.0253 1.1011

Factor5 | 0.01618 0.05895 0.0026 1.1038

Factor6 | -0.04278 0.01276 -0.0069 1.0968

Factor7 | -0.05554 0.02434 -0.0090 1.0878

Factor8 | -0.07988 0.03603 -0.0129 1.0749

Factor9 | -0.11590 0.05366 -0.0188 1.0561

Factor10 | -0.16956 0.00712 -0.0275 1.0286

Factor11 | -0.17668 . -0.0286 1.0000

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Factor loadings (pattern matrix) and unique variances

-------------------------------------------------

Variable | Factor1 Factor2 | Uniqueness

-------------+--------------------+--------------

acsi1 | 0.9025 -0.1251 | 0.1699

acsi2 | 0.8092 -0.1716 | 0.3157

acsi3 | 0.7651 -0.0687 | 0.4099

cuex1 | 0.5401 0.4227 | 0.5296

cuex2 | 0.5348 0.4233 | 0.5348

cuex3 | 0.3641 0.3118 | 0.7702

perq1 | 0.8659 0.0744 | 0.2446

perq2 | 0.8323 0.0064 | 0.3072

perq3 | 0.5643 0.0759 | 0.6758

perv1 | 0.8024 -0.2272 | 0.3046

perv2 | 0.7335 -0.2647 | 0.3920

-------------------------------------------------

. rotate, oblique promax

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 2

Rotation: oblique promax (Kaiser off) Number of params = 21

--------------------------------------------------------------------------

Factor | Variance Proportion Rotated factors are correlated

-------------+------------------------------------------------------------

Factor1 | 5.48049 0.8880

Factor2 | 3.52586 0.5713

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Rotated factor loadings (pattern matrix) and unique variances

-------------------------------------------------

Variable | Factor1 Factor2 | Uniqueness

-------------+--------------------+--------------

acsi1 | 0.8571 0.0859 | 0.1699

acsi2 | 0.8245 0.0045 | 0.3157

acsi3 | 0.6914 0.1184 | 0.4099

cuex1 | 0.0441 0.6586 | 0.5296

cuex2 | 0.0391 0.6579 | 0.5348

cuex3 | 0.0044 0.4767 | 0.7702

perq1 | 0.6391 0.3197 | 0.2446

perq2 | 0.6756 0.2279 | 0.3072

perq3 | 0.3907 0.2417 | 0.6758

perv1 | 0.8713 -0.0651 | 0.3046

perv2 | 0.8502 -0.1291 | 0.3920

-------------------------------------------------

Factor rotation matrix

--------------------------------

| Factor1 Factor2

-------------+------------------

Factor1 | 0.9773 0.7547

Factor2 | -0.2117 0.6560

--------------------------------

. estat structure

Structure matrix: correlations between variables and promax(3) rotated common factors

----------------------------------

Variable | Factor1 Factor2

-------------+--------------------

acsi1 | 0.9085 0.5990

acsi2 | 0.8272 0.4981

acsi3 | 0.7623 0.5324

cuex1 | 0.4384 0.6850

cuex2 | 0.4330 0.6813

cuex3 | 0.2898 0.4794

perq1 | 0.8305 0.7024

perq2 | 0.8121 0.6324

perq3 | 0.5355 0.4757

perv1 | 0.8323 0.4566

perv2 | 0.7729 0.3799

----------------------------------

When oblique rotation is used, the meaning of the term “factor loading” is ambiguous as it can refer to either structure coefficients or pattern coefficients, which are clearly different quantities as the example shows.

Now, let's consider the second question. In other words, what value should the loading be greater than to be considered a problematic 'cross-loading'? The first method to detect if cross-loading exists is absolute comparison. If the absolute value of the coefficient between an item and a factor is higher than an arbitrary cutoff point (e.g., .3, .4, .5), then the item is considered to be 'loaded' on the factor. If an item is 'loaded' on more than one factor, the item is considered to be 'cross-loaded'. For example, let's say the cutoff point is .4, which is a fairly commonly used cutoff. In the above structure matrix, all items are cross-loaded except for CUEX2 and CUEX3. CUEX3 is not loaded on any factor. Cross-loadings are less common in the pattern matrix and are observed only in ACSI1 and ACSI2. The problem with this method is that the choice of an cutoff is arbitrary and changing the cutoff changes the judgment of cross-loadings substantially. For example, if the cutoff point is .5, there is no cross-loading in the above pattern matrix, but ACSI2, ACSI3, and CUEX3 are not loaded on any factor.

The second method is relative comparison. In this comparison, an item is 'loaded' on a factor which it has highest loading on among all the factors. These coefficient values are shown in boldface in the table above. We can think of two rules related to cross-loadings. The first rule is what we call row comparison (Henseler et al., 2015). That is, 'for *an item*, if the absolute value of the coefficient between the item and the loaded factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between the item and any unloaded factors (e.g., ACSI-Factor2), the item is cross-loaded. ' However, according to the above definition the loaded coefficient value is the maximum value among the coefficient values of the same row, so that a cross-loading by this definition cannot occur. If you look at the table above, you can confirm that there is no cross-loading by this rule at all. Another variant of this rule is that the loading must be at least .2 or some other arbitrary number higher than any of the potential cross-loadings. According to this rule, for example all the ACSI items would cross-load on both factors regardless whether pattern or structure coefficients are inspected.

The second rule is what we call column comparison (Thompson, 1997). That is, 'for *a factor*, if the absolute value of the coefficient between the loaded item and the factor (e.g., ACSI1-Factor1) is less than the absolute value of the coefficient between any unloaded items and the factor (e.g., PERV1-Factor1), the item is cross-loaded. ' Even with this rule, there is no cross-loading in the above pattern matrix. However, in the structure matrix there are cross-loadings in CUEX1, CUEX2, CUEX3, and PERQ3. For example, the coefficient of CUEX1-Factor1 is .438, of which absolute value is smaller than that of the coefficient of .832 of PERV1-Factor1.

### Problems / Limitations

One disadvantage of this technique is the lack of consensus on exactly what cross-loadings are. More fundamentally, examining cross-loadings to assess discriminant validity does not fit the definition of discriminant validity. For further discussion, see the main text.

## Factor correlation (obtained from EFA)

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

As each variable name implies, our example is expected to include four factors. However, as a result of the usual procedure for determining the number of factors, it was determined to be two, which was used for simplicity in the previous example. One possible alternative is to analyze the number of factors fixed at four. The following table shows the results.

. factor acsi1-perv2, factors(4)

(obs=10,417)

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 4

Rotation: (unrotated) Number of params = 38

--------------------------------------------------------------------------

Factor | Eigenvalue Difference Proportion Cumulative

-------------+------------------------------------------------------------

Factor1 | 5.70778 5.06981 0.9248 0.9248

Factor2 | 0.63797 0.34400 0.1034 1.0282

Factor3 | 0.29397 0.13791 0.0476 1.0758

Factor4 | 0.15606 0.13988 0.0253 1.1011

Factor5 | 0.01618 0.05895 0.0026 1.1038

Factor6 | -0.04278 0.01276 -0.0069 1.0968

Factor7 | -0.05554 0.02434 -0.0090 1.0878

Factor8 | -0.07988 0.03603 -0.0129 1.0749

Factor9 | -0.11590 0.05366 -0.0188 1.0561

Factor10 | -0.16956 0.00712 -0.0275 1.0286

Factor11 | -0.17668 . -0.0286 1.0000

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Factor loadings (pattern matrix) and unique variances

---------------------------------------------------------------------

Variable | Factor1 Factor2 Factor3 Factor4 | Uniqueness

-------------+----------------------------------------+--------------

acsi1 | 0.9025 -0.1251 -0.0650 -0.0572 | 0.1624

acsi2 | 0.8092 -0.1716 -0.0816 -0.0260 | 0.3084

acsi3 | 0.7651 -0.0687 0.0052 -0.0448 | 0.4078

cuex1 | 0.5401 0.4227 0.1830 -0.0825 | 0.4893

cuex2 | 0.5348 0.4233 0.1401 -0.0345 | 0.5140

cuex3 | 0.3641 0.3118 -0.0354 0.2308 | 0.7157

perq1 | 0.8659 0.0744 -0.1217 -0.1316 | 0.2125

perq2 | 0.8323 0.0064 -0.2065 -0.0748 | 0.2590

perq3 | 0.5643 0.0759 -0.2325 0.2239 | 0.5716

perv1 | 0.8024 -0.2272 0.2167 0.0814 | 0.2510

perv2 | 0.7335 -0.2647 0.2651 0.0960 | 0.3125

---------------------------------------------------------------------

. rotate, oblique promax

Factor analysis/correlation Number of obs = 10,417

Method: principal factors Retained factors = 4

Rotation: oblique promax (Kaiser off) Number of params = 38

--------------------------------------------------------------------------

Factor | Variance Proportion Rotated factors are correlated

-------------+------------------------------------------------------------

Factor1 | 5.37323 0.8706

Factor2 | 3.79038 0.6142

Factor3 | 3.00720 0.4873

Factor4 | 2.84995 0.4618

--------------------------------------------------------------------------

LR test: independent vs. saturated: chi2(55) = 7.5e+04 Prob>chi2 = 0.0000

Rotated factor loadings (pattern matrix) and unique variances

---------------------------------------------------------------------

Variable | Factor1 Factor2 Factor3 Factor4 | Uniqueness

-------------+----------------------------------------+--------------

acsi1 | 0.7632 0.1948 0.0116 -0.0005 | 0.1624

acsi2 | 0.7029 0.2019 -0.0760 0.0282 | 0.3084

acsi3 | 0.5665 0.2087 0.0867 -0.0145 | 0.4078

cuex1 | 0.0898 0.0233 0.6744 -0.0424 | 0.4893

cuex2 | 0.0762 0.0076 0.6228 0.0388 | 0.5140

cuex3 | -0.1091 0.0189 0.2424 0.4192 | 0.7157

perq1 | 0.8037 -0.0407 0.1972 -0.0226 | 0.2125

perq2 | 0.8315 -0.0611 0.0400 0.0702 | 0.2590

perq3 | 0.3220 -0.0027 -0.0898 0.4603 | 0.5716

perv1 | 0.2934 0.6166 0.0253 0.0120 | 0.2510

perv2 | 0.1975 0.6820 0.0031 -0.0070 | 0.3125

---------------------------------------------------------------------

Factor rotation matrix

--------------------------------------------------

| Factor1 Factor2 Factor3 Factor4

-------------+------------------------------------

Factor1 | 0.9686 0.7989 0.6875 0.6872

Factor2 | -0.0809 -0.3643 0.6722 0.3504

Factor3 | -0.2023 0.4429 0.2636 -0.3095

Factor4 | -0.1198 0.1814 -0.0777 0.5561

--------------------------------------------------

. estat common

Correlation matrix of the promax(3) rotated common factors

------------------------------------------------------

Factors | Factor1 Factor2 Factor3 Factor4

-------------+----------------------------------------

Factor1 | 1

Factor2 | .6919 1

Factor3 | .5674 .407 1

Factor4 | .6332 .3852 .5832 1

------------------------------------------------------

### Problems / Limitations

EFA is a technique of 'letting the data speak', and the theoretical background and the intention of the researcher are not taken into consideration at all. Therefore, EFA often produces results that are difficult to interpret. The following shows the derived pattern matrix. Many items are 'loaded' on unintended factors. This discussion shows why CFA, not EFA, should be used when looking for factor correlation.

We will next describe techniques for evaluating discriminant validity using Stata’s SEM command.

## Factor correlation (point estimate)

### How often is it used?

AMJ 0%, JAP 1.4%, ORM 0%

### How to obtain

Two methods are used to determine the scale of the latent variable in SEM:

1. Fix one of the path coefficients between the latent variable and the corresponding manifest variables (i.e., factor loadings) to a non-zero value (typically 1), or
2. Fix the variance of latent variables to a non-zero value (typically 1).

The first style is the lavaan default, so it is used more often. However, the second style is more convenient for estimating factor correlations. We use the second style by using the variances option to constrain the latent variable variances to ones. The advantage of fixing the variance of latent variables to 1 is that the estimated factor covariance are correlations that can be interpreted directly without any further calculation. That is, the correlation table as shown in Table 41 can be obtained from the following results.

Table 45 Factor correlation estimates from Stata

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | .612 | 1.000 |  |  |
| PERQ | .957 | .698 | 1.000 |  |
| PERV | .875 | .526 | .770 | 1.000 |

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) // Alternative scaling

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -204840.68

Iteration 1: log likelihood = -204807.78

Iteration 2: log likelihood = -204807.45

Iteration 3: log likelihood = -204807.45

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -204807.45

( 1) [/]var(ACSI) = 1

( 2) [/]var(CUEX) = 1

( 3) [/]var(PERQ) = 1

( 4) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.852872 .015054 123.08 0.000 1.823366 1.882377

\_cons | -3.99e-09 .0195947 -0.00 1.000 -.0384048 .0384048

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.744797 .0169444 102.97 0.000 1.711587 1.778008

\_cons | -1.11e-08 .0205744 -0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.459294 .0159323 91.59 0.000 1.428067 1.490521

\_cons | -6.56e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.581242 .0205887 76.80 0.000 1.540888 1.621595

\_cons | 2.10e-10 .0205744 0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.646747 .0216043 76.22 0.000 1.604403 1.68909

\_cons | -2.57e-08 .0215541 -0.00 1.000 -.0422453 .0422453

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.037848 .0242034 42.88 0.000 .9904098 1.085285

\_cons | -3.46e-08 .0225339 -0.00 1.000 -.0441656 .0441655

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.710939 .0147078 116.33 0.000 1.682113 1.739766

\_cons | -1.03e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.563125 .0142597 109.62 0.000 1.535176 1.591073

\_cons | 2.50e-09 .0176352 0.00 1.000 -.0345643 .0345644

------------+----------------------------------------------------------------

perq3 |

PERQ | .9779162 .0156875 62.34 0.000 .9471692 1.008663

\_cons | -6.33e-09 .0166555 -0.00 1.000 -.0326441 .0326441

------------+----------------------------------------------------------------

perv1 |

PERV | 1.643444 .0142946 114.97 0.000 1.615427 1.671461

\_cons | -1.46e-08 .0176352 -0.00 1.000 -.0345644 .0345643

------------+----------------------------------------------------------------

perv2 |

PERV | 1.855681 .018127 102.37 0.000 1.820152 1.891209

\_cons | -2.47e-08 .0215541 -0.00 1.000 -.0422453 .0422453

--------------+----------------------------------------------------------------

var(e.acsi1)| .5664824 .012807 .5419292 .592148

var(e.acsi2)| 1.365259 .0218811 1.323039 1.408826

var(e.acsi3)| 1.480114 .0224018 1.436853 1.524679

var(e.cuex1)| 1.909252 .0436581 1.825573 1.996767

var(e.cuex2)| 2.127761 .0480061 2.035721 2.223962

var(e.cuex3)| 4.212365 .0629737 4.090729 4.337617

var(e.perq1)| .6823401 .0144948 .6545141 .711349

var(e.perq2)| .7963304 .0146877 .7680571 .8256445

var(e.perq3)| 1.933403 .0278122 1.879653 1.988689

var(e.perv1)| .5387805 .0174338 .5056719 .5740568

var(e.perv2)| 1.395985 .0278857 1.342386 1.451724

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .6124711 .0085465 71.66 0.000 .5957203 .629222

cov(ACSI,PERQ)| .9569207 .0024519 390.28 0.000 .9521151 .9617263

cov(ACSI,PERV)| .8746049 .0038616 226.49 0.000 .8670364 .8821734

cov(CUEX,PERQ)| .6983861 .0077252 90.40 0.000 .683245 .7135273

cov(CUEX,PERV)| .5260119 .0095995 54.80 0.000 .5071973 .5448265

cov(PERQ,PERV)| .7695618 .0054534 141.12 0.000 .7588734 .7802502

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(38) = 1614.29, Prob > chi2 = 0.0000

. estimates store m1

### Problems / Limitations

The point estimate provides only limited information about the parameter. Particularly, a single point estimate does not tell us anything about how certain we are about the estimate. A better alternative is an interval estimate in the form of a 95% confidence interval, which gives information on the maximum and minimum values of the parameter when the assumptions are met at a given confidence level.

## Factor correlation (whether the confidence interval includes 1)

### How often is it used?

AMJ 0%, JAP 0%, ORM 5.0%

### How to obtain

Confidence intervals are included in the default sem output shown above.

Table 46 Confidence intervals for correlations obtained from Stata

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ACSI | CUEX | PERQ | PERV |
| ACSI | 1.000 |  |  |  |
| CUEX | [.596,.629] | 1.000 |  |  |
| PERQ | [.952,.962] | [.683,.713] | 1.000 |  |
| PERV | [.867,.882] | [.507,.545] | [.759,.780] | 1.000 |

### Problems / Limitations

There is no problem with this technique itself, but the problem lies in the way we have used this technique this far. When evaluating discriminant validity, we have examined whether the interval estimates of factor correlation include one (i.e., perfect correlation). That is, if the maximum value of the confidence interval is less than 1, it is determined that there is no discriminant validity problem. This is problematic because almost all data will meet these criteria as long as the sample size is large enough. For example, in the above example, the factor correlation between ACSI-PERQ is very high, but its confidence interval does not include 1.

## Techniques using model fit indices: no comparison

### How often is it used?

AMJ 11.1%, JAP 1.4%, ORM 0%

### How to obtain

The values of the model fit indices are automatically calculated by Stata. The statistic is reported by default and more fit indices can be obtained by using estat gof postestimation command. For example, this case the fit indices are = 1614.286, p < 0.001, TLI 970, CFI .979, and RMSEA .063. The value shows that the model does not fit exactly. While there are SEM model evaluation guidelines that provide cutoffs for the other indices and our values would be considered acceptable against these cutoffs, we nevertheless suggest that researchers diagnose their models to understand the source of misfit before declaring misfit acceptable (Kline, 2011, Chapter 8). However, applying this technique makes no indication of any problem in the discriminant validity of these data.

. estat gof, stats(all)

----------------------------------------------------------------------------

Fit statistic | Value Description

---------------------+------------------------------------------------------

Likelihood ratio |

chi2\_ms(38) | 1614.286 model vs. saturated

p > chi2 | 0.000

chi2\_bs(55) | 75026.313 baseline vs. saturated

p > chi2 | 0.000

---------------------+------------------------------------------------------

Population error |

RMSEA | 0.063 Root mean squared error of approximation

90% CI, lower bound | 0.060

upper bound | 0.066

pclose | 0.000 Probability RMSEA <= 0.05

---------------------+------------------------------------------------------

Information criteria |

AIC | 409692.906 Akaike's information criterion

BIC | 409975.703 Bayesian information criterion

---------------------+------------------------------------------------------

Baseline comparison |

CFI | 0.979 Comparative fit index

TLI | 0.970 Tucker-Lewis index

---------------------+------------------------------------------------------

Size of residuals |

SRMR | 0.029 Standardized root mean squared residual

CD | 0.995 Coefficient of determination

----------------------------------------------------------------------------

### Problems / Limitations

The fit of the proposed model has nothing to do with the discriminant validity. Assessing discriminant validity requires a well-fitting model, but the model fit itself does not inform us about discriminant validity. To assess discriminant validity using model fit indices, a comparison with other alternative models is needed. The question is which alternative model to compare.

## Techniques using model fit indices: compared to nested models with fewer factors

### How often is it used?

AMJ 29.6%, JAP 58.9%, ORM 25.0%

As far as we know, there are no guidelines-type article that recommends the use of this technique for evaluating discriminant validity. Surprisingly, however, this technique is the most commonly used technique in applied psychology.

### How to obtain

Because there is no authoritative source, this technique is being applied in a wide variety of ways. A typical method is as follows. Suppose the proposed model is composed of N factors. Then, we can construct (N - 1) -factor models, (N - 2)-factor models … and a 1-factor model by merging some of the factors into one. This technique compares the fit indices of all these (or some arbitrarily selected) alternative models with the originally proposed models.

In our ACSI example, the factor correlation between ACSI and PERQ is the highest, so you can compare an alternative three-factor model that combines the two factors into one factor with the originally proposed model. (Of course, it is also possible to review comparisons with all possible three-factor, two-factor, and one-factor models.) The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

. sem (ACSIPERQ -> acsi1 acsi2 acsi3 perq1 perq2 perq3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERV -> perv1 perv2), ///

variance(ACSIPERQ@1 CUEX@1 PERV@1) // Alternative scaling

Endogenous variables

Measurement: acsi1 acsi2 acsi3 perq1 perq2 perq3 cuex1 cuex2 cuex3 perv1 perv2

Exogenous variables

Latent: ACSIPERQ CUEX PERV

Fitting target model:

Iteration 0: log likelihood = -205519.6

Iteration 1: log likelihood = -205486.82

Iteration 2: log likelihood = -205486.57

Iteration 3: log likelihood = -205486.57

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -205486.57

( 1) [/]var(ACSIPERQ) = 1

( 2) [/]var(CUEX) = 1

( 3) [/]var(PERV) = 1

-----------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

------------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSIPERQ | 1.844379 .0150666 122.42 0.000 1.814849 1.873909

\_cons | -3.99e-09 .0195947 -0.00 1.000 -.0384048 .0384048

----------------+----------------------------------------------------------------

acsi2 |

ACSIPERQ | 1.733462 .0169901 102.03 0.000 1.700162 1.766762

\_cons | -1.11e-08 .0205744 -0.00 1.000 -.0403251 .0403251

----------------+----------------------------------------------------------------

acsi3 |

ACSIPERQ | 1.458043 .015935 91.50 0.000 1.426811 1.489275

\_cons | -6.56e-09 .0186149 -0.00 1.000 -.0364846 .0364846

----------------+----------------------------------------------------------------

perq1 |

ACSIPERQ | 1.660874 .0148684 111.71 0.000 1.631732 1.690015

\_cons | -1.03e-09 .0186149 -0.00 1.000 -.0364846 .0364846

----------------+----------------------------------------------------------------

perq2 |

ACSIPERQ | 1.528939 .0143352 106.66 0.000 1.500842 1.557035

\_cons | 2.50e-09 .0176352 0.00 1.000 -.0345643 .0345644

----------------+----------------------------------------------------------------

perq3 |

ACSIPERQ | .9583224 .0156416 61.27 0.000 .9276654 .9889793

\_cons | -6.33e-09 .0166555 -0.00 1.000 -.0326441 .0326441

----------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.573933 .0208003 75.67 0.000 1.533165 1.6147

\_cons | 2.10e-10 .0205744 0.00 1.000 -.0403251 .0403251

----------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.656293 .0218099 75.94 0.000 1.613547 1.69904

\_cons | -2.57e-08 .0215541 -0.00 1.000 -.0422453 .0422453

----------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.034948 .024286 42.62 0.000 .9873483 1.082548

\_cons | -3.46e-08 .0225339 -0.00 1.000 -.0441656 .0441655

----------------+----------------------------------------------------------------

perv1 |

PERV | 1.662388 .0142874 116.35 0.000 1.634385 1.690391

\_cons | -1.46e-08 .0176352 -0.00 1.000 -.0345644 .0345643

----------------+----------------------------------------------------------------

perv2 |

PERV | 1.834534 .0182875 100.32 0.000 1.798691 1.870377

\_cons | -2.47e-08 .0215541 -0.00 1.000 -.0422453 .0422453

------------------+----------------------------------------------------------------

var(e.acsi1)| .5978837 .0124401 .573992 .6227698

var(e.acsi2)| 1.404687 .0222279 1.36179 1.448936

var(e.acsi3)| 1.483765 .0224282 1.440452 1.528382

var(e.perq1)| .8511516 .0146918 .8228378 .8804396

var(e.perq2)| .9020348 .0147946 .8734991 .9315027

var(e.perq3)| 1.971341 .0280671 1.917091 2.027126

var(e.cuex1)| 1.932313 .0445839 1.846876 2.021701

var(e.cuex2)| 2.096227 .0490946 2.002179 2.194694

var(e.cuex3)| 4.218375 .0631523 4.096397 4.343985

var(e.perv1)| .4761545 .0181233 .4419261 .5130341

var(e.perv2)| 1.474021 .0289781 1.418306 1.531926

var(ACSIPERQ)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERV)| 1 (constrained)

------------------+----------------------------------------------------------------

cov(ACSIPERQ,CUEX)| .6553621 .0078577 83.40 0.000 .6399614 .6707629

cov(ACSIPERQ,PERV)| .8392103 .0041406 202.68 0.000 .8310949 .8473257

cov(CUEX,PERV)| .5243691 .0095683 54.80 0.000 .5056155 .5431227

-----------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(41) = 2972.52, Prob > chi2 = 0.0000

. estimates store m2

. lrtest m1 m2

Likelihood-ratio test LR chi2(3) = 1358.24

(Assumption: m2 nested in m1) Prob > chi2 = 0.0000

The correlation between ACSI and PERQ was so high that merging them into one factor seemed like a good idea, but the 3-factor model showed inferior fit indices to the original 4-factor model.

### Problems / Limitations

Notice that the difference in degrees of freedom between the proposed model and the alternative model is three. Originally, our interest was a high correlation between ACSI and PERQ, but merging the two factors into one adds additional constraints. In other words, the above model is equivalent to the original model with the following three constraints.

1. The correlation between ACSI and PERQ (.957) is 1.
2. The correlation between ACSI and CUEX (.612) and the correlation between PERQ and CUEX (.698) are equal.
3. The correlation between ACSI and PERV (.875) and the correlation between PERQ and PERV (. 770) are equal.

That is, the following model:

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) /// Alternative scaling

covariance(ACSI\*PERQ@1 ACSI\*CUEX@a PERQ\*CUEX@a ACSI\*PERV@b PERQ\*PERV@b)

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -205717.27

Iteration 1: log likelihood = -205499.74

Iteration 2: log likelihood = -205486.59

Iteration 3: log likelihood = -205486.57

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -205486.57

( 1) [/]var(ACSI) = 1

( 2) [/]cov(ACSI,CUEX) - [/]cov(CUEX,PERQ) = 0

( 3) [/]cov(ACSI,PERQ) = 1

( 4) [/]cov(ACSI,PERV) - [/]cov(PERQ,PERV) = 0

( 5) [/]var(CUEX) = 1

( 6) [/]var(PERQ) = 1

( 7) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.844378 .0150666 122.42 0.000 1.814848 1.873908

\_cons | -3.99e-09 .0195947 -0.00 1.000 -.0384048 .0384048

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.733461 .0169901 102.03 0.000 1.700161 1.766761

\_cons | -1.11e-08 .0205744 -0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.458042 .015935 91.50 0.000 1.42681 1.489274

\_cons | -6.56e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.573931 .0208003 75.67 0.000 1.533163 1.614699

\_cons | 2.10e-10 .0205744 0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.656295 .0218099 75.94 0.000 1.613548 1.699041

\_cons | -2.57e-08 .0215541 -0.00 1.000 -.0422453 .0422453

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.034949 .024286 42.62 0.000 .9873495 1.082549

\_cons | -3.46e-08 .0225339 -0.00 1.000 -.0441656 .0441655

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.660873 .0148683 111.71 0.000 1.631732 1.690015

\_cons | -1.03e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.528938 .0143352 106.66 0.000 1.500842 1.557035

\_cons | 2.50e-09 .0176352 0.00 1.000 -.0345643 .0345643

------------+----------------------------------------------------------------

perq3 |

PERQ | .9583219 .0156416 61.27 0.000 .927665 .9889788

\_cons | -6.33e-09 .0166555 -0.00 1.000 -.0326441 .0326441

------------+----------------------------------------------------------------

perv1 |

PERV | 1.662388 .0142874 116.35 0.000 1.634386 1.690391

\_cons | -1.46e-08 .0176352 -0.00 1.000 -.0345644 .0345643

------------+----------------------------------------------------------------

perv2 |

PERV | 1.834533 .0182874 100.32 0.000 1.79869 1.870376

\_cons | -2.47e-08 .0215541 -0.00 1.000 -.0422453 .0422453

--------------+----------------------------------------------------------------

var(e.acsi1)| .5978827 .0124401 .5739911 .6227687

var(e.acsi2)| 1.404688 .0222279 1.36179 1.448936

var(e.acsi3)| 1.483765 .0224282 1.440452 1.528382

var(e.cuex1)| 1.932318 .0445839 1.846881 2.021707

var(e.cuex2)| 2.096221 .0490945 2.002173 2.194687

var(e.cuex3)| 4.218372 .0631522 4.096394 4.343982

var(e.perq1)| .8511508 .0146918 .8228371 .8804388

var(e.perq2)| .902034 .0147945 .8734984 .9315019

var(e.perq3)| 1.971341 .0280671 1.917091 2.027126

var(e.perv1)| .4761529 .0181233 .4419245 .5130324

var(e.perv2)| 1.474023 .0289781 1.418307 1.531927

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .655363 .0078576 83.40 0.000 .6399623 .6707637

cov(ACSI,PERQ)| 1 (constrained)

cov(ACSI,PERV)| .8392106 .0041406 202.68 0.000 .8310952 .847326

cov(CUEX,PERQ)| .655363 .0078576 83.40 0.000 .6399623 .6707637

cov(CUEX,PERV)| .5243699 .0095683 54.80 0.000 .5056163 .5431234

cov(PERQ,PERV)| .8392106 .0041406 202.68 0.000 .8310952 .847326

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(41) = 2972.52, Prob > chi2 = 0.0000

. estimates store m3

. lrtest m1 m3

Likelihood-ratio test LR chi2(3) = 1358.24

(Assumption: m3 nested in m1) Prob > chi2 = 0.0000

////////// against model with correlation fixed at 1 //////////

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) /// Alternative scaling

covariance(ACSI\*PERQ@1)

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -205392.17

Iteration 1: log likelihood = -205031.73

Iteration 2: log likelihood = -205000.56

Iteration 3: log likelihood = -205000.2

Iteration 4: log likelihood = -205000.2

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -205000.2

( 1) [/]var(ACSI) = 1

( 2) [/]cov(ACSI,PERQ) = 1

( 3) [/]var(CUEX) = 1

( 4) [/]var(PERQ) = 1

( 5) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.832158 .0151315 121.08 0.000 1.802501 1.861815

\_cons | -3.99e-09 .0195947 -0.00 1.000 -.0384048 .0384048

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.725939 .017014 101.44 0.000 1.692592 1.759285

\_cons | -1.11e-08 .0205744 -0.00 1.000 -.0403251 .0403251

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.446965 .0159458 90.74 0.000 1.415712 1.478218

\_cons | -6.56e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.575142 .0205338 76.71 0.000 1.534896 1.615387

\_cons | 2.10e-10 .0205461 0.00 1.000 -.0402697 .0402697

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.644124 .0215569 76.27 0.000 1.601873 1.686375

\_cons | -2.57e-08 .0215247 -0.00 1.000 -.0421877 .0421877

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.037211 .0241407 42.97 0.000 .989896 1.084526

\_cons | -3.46e-08 .0225227 -0.00 1.000 -.0441437 .0441436

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.678133 .0147812 113.53 0.000 1.649163 1.707104

\_cons | -1.03e-09 .0186149 -0.00 1.000 -.0364846 .0364846

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.537217 .0143298 107.27 0.000 1.509131 1.565303

\_cons | 2.50e-09 .0176352 0.00 1.000 -.0345643 .0345644

------------+----------------------------------------------------------------

perq3 |

PERQ | .9643831 .0156345 61.68 0.000 .9337401 .9950261

\_cons | -6.33e-09 .0166555 -0.00 1.000 -.0326441 .0326441

------------+----------------------------------------------------------------

perv1 |

PERV | 1.638234 .0142651 114.84 0.000 1.610275 1.666194

\_cons | -1.46e-08 .0175909 -0.00 1.000 -.0344776 .0344776

------------+----------------------------------------------------------------

perv2 |

PERV | 1.85038 .0180409 102.57 0.000 1.815021 1.88574

\_cons | -2.47e-08 .0215079 -0.00 1.000 -.0421548 .0421547

--------------+----------------------------------------------------------------

var(e.acsi1)| .6428144 .0127147 .618371 .6682241

var(e.acsi2)| 1.430713 .0224064 1.387464 1.475309

var(e.acsi3)| 1.515945 .0225897 1.47231 1.560873

var(e.cuex1)| 1.916396 .0436584 1.832709 2.003904

var(e.cuex2)| 2.1232 .0480227 2.031133 2.21944

var(e.cuex3)| 4.208435 .0629504 4.086846 4.333642

var(e.perq1)| .7935225 .0141878 .7661964 .8218231

var(e.perq2)| .8766533 .0148647 .8479977 .9062772

var(e.perq3)| 1.959688 .0279419 1.905681 2.015225

var(e.perv1)| .5396303 .0174338 .50652 .5749049

var(e.perv2)| 1.394901 .0278793 1.341315 1.450628

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .6284717 .0083231 75.51 0.000 .6121586 .6447847

cov(ACSI,PERQ)| 1 (constrained)

cov(ACSI,PERV)| .8783835 .0038648 227.28 0.000 .8708087 .8859583

cov(CUEX,PERQ)| .7017722 .0077856 90.14 0.000 .6865127 .7170316

cov(CUEX,PERV)| .534371 .0094612 56.48 0.000 .5158273 .5529147

cov(PERQ,PERV)| .7967475 .0048869 163.04 0.000 .7871693 .8063257

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(39) = 1999.77, Prob > chi2 = 0.0000

. estimates store m4

. lrtest m1 m4

Likelihood-ratio test LR chi2(1) = 385.49

(Assumption: m4 nested in m1) Prob > chi2 = 0.0000

Of these, only the first constraint is truly relevant to discriminant validity, and a strategy that focuses only on the necessary constraints is needed.

## Techniques using model fit indices: comparison against model with correlation fixed at 1

### How often is it used?

AMJ 14.8%, JAP 1.4%, ORM 10.0%

### How to obtain

For all possible latent variable pairs, the model with the correlation fixed at 1 is compared with the original model. Here, we present only the model with the correlation between ACSI and PERQ is constrained to 1.

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) /// Alternative scaling

covariance(ACSI\*PERQ@.96)

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -204851.01

Iteration 1: log likelihood = -204808.66

Iteration 2: log likelihood = -204808.26

Iteration 3: log likelihood = -204808.26

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -204808.26

( 1) [/]var(ACSI) = 1

( 2) [/]cov(ACSI,PERQ) = .96

( 3) [/]var(CUEX) = 1

( 4) [/]var(PERQ) = 1

( 5) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.855401 .014992 123.76 0.000 1.826017 1.884785

\_cons | -3.99e-09 .0196268 -0.00 1.000 -.0384678 .0384678

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.747325 .0168918 103.44 0.000 1.714217 1.780432

\_cons | -1.11e-08 .0206015 -0.00 1.000 -.0403783 .0403783

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.461616 .0158785 92.05 0.000 1.430494 1.492737

\_cons | -6.56e-09 .0186359 -0.00 1.000 -.0365257 .0365257

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.582205 .0205953 76.82 0.000 1.541839 1.622571

\_cons | 2.10e-10 .0205825 0.00 1.000 -.040341 .040341

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.647994 .0216037 76.28 0.000 1.605652 1.690337

\_cons | -2.57e-08 .0215626 -0.00 1.000 -.0422619 .0422618

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.038665 .0242149 42.89 0.000 .9912046 1.086125

\_cons | -3.46e-08 .0225371 -0.00 1.000 -.0441719 .0441718

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.712683 .0147109 116.42 0.000 1.68385 1.741516

\_cons | -1.03e-09 .0186438 -0.00 1.000 -.0365411 .0365411

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.565172 .014228 110.01 0.000 1.537285 1.593058

\_cons | 2.50e-09 .0176606 0.00 1.000 -.0346141 .0346142

------------+----------------------------------------------------------------

perq3 |

PERQ | .9793535 .0156838 62.44 0.000 .9486139 1.010093

\_cons | -6.33e-09 .016666 -0.00 1.000 -.0326648 .0326648

------------+----------------------------------------------------------------

perv1 |

PERV | 1.645257 .0142592 115.38 0.000 1.61731 1.673205

\_cons | -1.46e-08 .0176516 -0.00 1.000 -.0345965 .0345964

------------+----------------------------------------------------------------

perv2 |

PERV | 1.857766 .0180905 102.69 0.000 1.822309 1.893222

\_cons | -2.47e-08 .0215712 -0.00 1.000 -.0422788 .0422788

--------------+----------------------------------------------------------------

var(e.acsi1)| .5702313 .0124605 .5463247 .595184

var(e.acsi2)| 1.368076 .021786 1.326036 1.411449

var(e.acsi3)| 1.481479 .0223753 1.438267 1.52599

var(e.cuex1)| 1.909693 .0436568 1.826016 1.997204

var(e.cuex2)| 2.127436 .0480065 2.035395 2.223638

var(e.cuex3)| 4.212171 .0629725 4.090538 4.337421

var(e.perq1)| .6875572 .0138962 .6608535 .7153399

var(e.perq2)| .7992684 .0144988 .7713505 .8281966

var(e.perq3)| 1.934247 .0278043 1.880512 1.989517

var(e.perv1)| .5388349 .0174335 .5057267 .5741107

var(e.perv2)| 1.395916 .0278851 1.342318 1.451653

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .6145338 .0083698 73.42 0.000 .5981292 .6309384

cov(ACSI,PERQ)| .96 (constrained)

cov(ACSI,PERV)| .8751204 .0038323 228.35 0.000 .8676092 .8826316

cov(CUEX,PERQ)| .6990885 .0077028 90.76 0.000 .6839914 .7141856

cov(CUEX,PERV)| .5276878 .009491 55.60 0.000 .5090858 .5462898

cov(PERQ,PERV)| .7721496 .0050171 153.90 0.000 .7623162 .7819829

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(39) = 1615.89, Prob > chi2 = 0.0000

. estimates store m5

. lrtest m1 m5

Likelihood-ratio test LR chi2(1) = 1.61

(Assumption: m5 nested in m1) Prob > chi2 = 0.2046

Although the correlation between ACSI and PERQ is very high at .957, the model with a fixed correlation of 1 has poorer fit indices than the original model.

### Problems / Limitations

There are no logical flaws in this technique. However, there is a practical problem that almost all data pass the criteria as long as the sample size is large enough because correlations are rarely exactly 1. Thus, applied researchers have preferred a technique that require more difficult-to-pass criteria.

## AVE: compared with the square of factor correlation

### How often is it used?

AMJ 7.4%, JAP 5.5%, ORM 5.0%

Although this technique is not used very often among organizational researchers, it is a standard technique for evaluating discriminant validity in many other business disciplines, such as marketing.

### How to obtain

The original formula of Average Variance Extracted (AVE), proposed by Fornell and Larcker (1981) is:

where is the standardized loading of indicator . Because of standardization, this can be simplified to be the mean of squared factor loadings.

For example, let's calculate ACSI’s AVE using this simpler formula:

If we use the original formula and the unstandardized coefficients, the resulting AVE will be different:

As can be seen from the above calculations, the two versions of the AVE formula usually yield close values, but are not mathematically equivalent. Instead of a mathematical proof of how the two formulas differ, we present a simple analogy. For example, assume that there are three values , , and . The original formula is to calculate , and the formula using the standardized coefficients is to calculate ++) . For the same reason, the value obtained by applying the standardized coefficients to the original formula of AVE is different from the value obtained by applying the unstandardized coefficients to the same formula. Therefore, when presenting the AVE value, we should specify whether standardized or unstandardized coefficients are used. However, note that the interpretation of AVE as variance explained is only valid if the latent variables were scaled to unit variances (which would be automatically the case in fully standardized estimates).

. scalar ave1 = (loadings[1,1]^2 + loadings[2,1]^2 + loadings[3,1]^2) / ///

(loadings[1,1]^2 + loadings[2,1]^2 + loadings[3,1]^2 + ///

errors[1,1] + errors[2,2] + errors[3,3])

. scalar ave2 = (loadings[4,2]^2 + loadings[5,2]^2 + loadings[6,2]^2) / ///

(loadings[4,2]^2 + loadings[5,2]^2 + loadings[6,2]^2 + ///

errors[4,4] + errors[5,5] + errors[6,6])

. scalar ave3 = (loadings[7,3]^2 + loadings[8,3]^2 + loadings[9,3]^2) / ///

(loadings[7,3]^2 + loadings[8,3]^2 + loadings[9,3]^2 + ///

errors[7,7] + errors[8,8] + errors[9,9])

. scalar ave4 = (loadings[10,4]^2 + loadings[11,4]^2) / ///

(loadings[10,4]^2 + loadings[11,4]^2 + ///

errors[10,10] + errors[11,11])

We will now explain how the AVE values are used in the The Fornell-Larcker criterion for assessing discriminant validity. We will use Stata’s mata matrix algebra tools to raise all elements of the estimated factor correlation matrix to second power and then set the diagonal to the AVE values to produce a comparison matrix.

. mata : st\_matrix("comparisonMatrix", st\_matrix("factorCorrelations") :^2)

. matrix comparisonMatrix[1,1] = ave1

. matrix comparisonMatrix[2,2] = ave2

. matrix comparisonMatrix[3,3] = ave3

. matrix comparisonMatrix[4,4] = ave4

. matrix list comparisonMatrix

symmetric comparisonMatrix[4,4]

c1 c2 c3 c4

r1 .71612453

r2 .3751209 .4325881

r3 .91569718 .4877432 .64965086

r4 .76493373 .27668851 .59222535 .76052582

The Fornell-Larcker criterion for assessing discriminant validity is that for every pair of latent variables, the square of the factor correlation must be less than the AVE values of both latent variables. The acronym AVE/SV comes from the fact that the square of the factor correlation is also called shared variance. The above example fails this criterion for three pairs of latent variables: 1) The shared variance between ACSI and PERQ is 91569718, which is larger than ACSI's AVE value of .71612453 and PERQ's AVE value of .64965086) The shared variance between ACSI and PERV is .76493373, which is larger than the ACSI’s AVE value of 71612453. 3) The shared variance between CUEX and PERQ is .4877432, which is greater than the CUEX’s AVE value of .4325881.

### Problems / Limitations

Few methodological studies have seriously considered this technique, and the logical problems of this technique are almost unknown. Essentially, AVE is a weighted average of item-level reliabilities. While factor correlation is related to discriminant validity, as a comparison point, AVE has little relevance. Moreover, given methodological literature’s lack of attention to this technique, it is often is misapplied in a variety of ways. The most common misuse is that AVE is compared to a value other than the square of the factor correlation.

## AVE: compared with the square of scale score correlation

### How often is it used?

AMJ 3.7%, JAP 1.4%, ORM 0%

Many users are more familiar with the process of obtaining scale score correlations than factor correlations. Therefore, unlike the original proposal of Fornell and Larcker (1981a), studies comparing AVE to the square of the scale score correlation are often found.

### How to obtain

The procedure for constructing the comparison table is identical to the previous example except that we use the scale score correlation matrix.

. correlate acsi cuex perq perv

(obs=10,417)

| acsi cuex perq perv

-------------+------------------------------------

acsi | 1.0000

cuex | 0.4858 1.0000

perq | 0.8171 0.5497 1.0000

perv | 0.7645 0.4041 0.6446 1.0000

. matrix scaleScoreCorrelations = r(C)

. mata : st\_matrix("comparisonMatrix", st\_matrix("scaleScoreCorrelations") :^2)

. matrix comparisonMatrix[1,1] = ave1

. matrix comparisonMatrix[2,2] = ave2

. matrix comparisonMatrix[3,3] = ave3

. matrix comparisonMatrix[4,4] = ave4

. matrix list comparisonMatrix

symmetric comparisonMatrix[4,4]

c1 c2 c3 c4

r1 .71612453

r2 .23603956 .4325881

r3 .6676081 .30218859 .64965086

r4 .58443505 .16330898 .4154539 .76052582

The Fornell-Larcker criterion was violated in only one case. The square of scale score correlation between ACSI and PERQ is .6676081, which is larger than PERQ's AVE value of .64965086.

### Problems / Limitations

It is clear that this technique is misuse, so much explanation is not necessary.

## Techniques using model fit indices: comparison against model with correlation fixed at cutoff point less than 1

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

### How to obtain

The fact that a correlation is not exactly 1 may be a necessary condition for discriminant validity, but it is difficult to consider it a sufficient condition. In other words, a more realistic test would test against a high but not necessarily perfect correlation. This is the idea in using references values below 1. Which cutoff to use is a matter of subjective judgment, but we can consider candidates such as .85, .9, or .95. This test makes sense only if the correlation estimate is below the the cutoff; if it does not, the discriminant validity test should be considered as failed. To demonstrate this test, we compare the correlation between ACSI and PERV against a fixed cutoff of .96, which in this case is just an arbitrarily chosen cutoff that is greater than the estimated correlation of .957. That is, we compare the following model with the original model.

. sem (ACSI -> acsi1 acsi2 acsi3) ///

(CUEX -> cuex1 cuex2 cuex3) ///

(PERQ -> perq1 perq2 perq3) ///

(PERV -> perv1 perv2), ///

variance(ACSI@1 CUEX@1 PERQ@1 PERV@1) /// Alternative scaling

covariance(ACSI\*PERQ@.96)

Endogenous variables

Measurement: acsi1 acsi2 acsi3 cuex1 cuex2 cuex3 perq1 perq2 perq3 perv1 perv2

Exogenous variables

Latent: ACSI CUEX PERQ PERV

Fitting target model:

Iteration 0: log likelihood = -204851.01

Iteration 1: log likelihood = -204808.66

Iteration 2: log likelihood = -204808.26

Iteration 3: log likelihood = -204808.26

Structural equation model Number of obs = 10,417

Estimation method = ml

Log likelihood = -204808.26

( 1) [/]var(ACSI) = 1

( 2) [/]cov(ACSI,PERQ) = .96

( 3) [/]var(CUEX) = 1

( 4) [/]var(PERQ) = 1

( 5) [/]var(PERV) = 1

-------------------------------------------------------------------------------

| OIM

| Coef. Std. Err. z P>|z| [95% Conf. Interval]

--------------+----------------------------------------------------------------

Measurement |

acsi1 |

ACSI | 1.855401 .014992 123.76 0.000 1.826017 1.884785

\_cons | -3.99e-09 .0196268 -0.00 1.000 -.0384678 .0384678

------------+----------------------------------------------------------------

acsi2 |

ACSI | 1.747325 .0168918 103.44 0.000 1.714217 1.780432

\_cons | -1.11e-08 .0206015 -0.00 1.000 -.0403783 .0403783

------------+----------------------------------------------------------------

acsi3 |

ACSI | 1.461616 .0158785 92.05 0.000 1.430494 1.492737

\_cons | -6.56e-09 .0186359 -0.00 1.000 -.0365257 .0365257

------------+----------------------------------------------------------------

cuex1 |

CUEX | 1.582205 .0205953 76.82 0.000 1.541839 1.622571

\_cons | 2.10e-10 .0205825 0.00 1.000 -.040341 .040341

------------+----------------------------------------------------------------

cuex2 |

CUEX | 1.647994 .0216037 76.28 0.000 1.605652 1.690337

\_cons | -2.57e-08 .0215626 -0.00 1.000 -.0422619 .0422618

------------+----------------------------------------------------------------

cuex3 |

CUEX | 1.038665 .0242149 42.89 0.000 .9912046 1.086125

\_cons | -3.46e-08 .0225371 -0.00 1.000 -.0441719 .0441718

------------+----------------------------------------------------------------

perq1 |

PERQ | 1.712683 .0147109 116.42 0.000 1.68385 1.741516

\_cons | -1.03e-09 .0186438 -0.00 1.000 -.0365411 .0365411

------------+----------------------------------------------------------------

perq2 |

PERQ | 1.565172 .014228 110.01 0.000 1.537285 1.593058

\_cons | 2.50e-09 .0176606 0.00 1.000 -.0346141 .0346142

------------+----------------------------------------------------------------

perq3 |

PERQ | .9793535 .0156838 62.44 0.000 .9486139 1.010093

\_cons | -6.33e-09 .016666 -0.00 1.000 -.0326648 .0326648

------------+----------------------------------------------------------------

perv1 |

PERV | 1.645257 .0142592 115.38 0.000 1.61731 1.673205

\_cons | -1.46e-08 .0176516 -0.00 1.000 -.0345965 .0345964

------------+----------------------------------------------------------------

perv2 |

PERV | 1.857766 .0180905 102.69 0.000 1.822309 1.893222

\_cons | -2.47e-08 .0215712 -0.00 1.000 -.0422788 .0422788

--------------+----------------------------------------------------------------

var(e.acsi1)| .5702313 .0124605 .5463247 .595184

var(e.acsi2)| 1.368076 .021786 1.326036 1.411449

var(e.acsi3)| 1.481479 .0223753 1.438267 1.52599

var(e.cuex1)| 1.909693 .0436568 1.826016 1.997204

var(e.cuex2)| 2.127436 .0480065 2.035395 2.223638

var(e.cuex3)| 4.212171 .0629725 4.090538 4.337421

var(e.perq1)| .6875572 .0138962 .6608535 .7153399

var(e.perq2)| .7992684 .0144988 .7713505 .8281966

var(e.perq3)| 1.934247 .0278043 1.880512 1.989517

var(e.perv1)| .5388349 .0174335 .5057267 .5741107

var(e.perv2)| 1.395916 .0278851 1.342318 1.451653

var(ACSI)| 1 (constrained)

var(CUEX)| 1 (constrained)

var(PERQ)| 1 (constrained)

var(PERV)| 1 (constrained)

--------------+----------------------------------------------------------------

cov(ACSI,CUEX)| .6145338 .0083698 73.42 0.000 .5981292 .6309384

cov(ACSI,PERQ)| .96 (constrained)

cov(ACSI,PERV)| .8751204 .0038323 228.35 0.000 .8676092 .8826316

cov(CUEX,PERQ)| .6990885 .0077028 90.76 0.000 .6839914 .7141856

cov(CUEX,PERV)| .5276878 .009491 55.60 0.000 .5090858 .5462898

cov(PERQ,PERV)| .7721496 .0050171 153.90 0.000 .7623162 .7819829

-------------------------------------------------------------------------------

LR test of model vs. saturated: chi2(39) = 1615.89, Prob > chi2 = 0.0000

.

. estimates store m5

.

. lrtest m1 m5

Likelihood-ratio test LR chi2(1) = 1.61

(Assumption: m5 nested in m1) Prob > chi2 = 0.2046

If the reference point is .96, the correlation between ACSI and PERV is not significantly less than that, so we can assess that there is a problem in the discriminant validity.

### Problems / Limitations

This technique has the advantage of being more flexible than the previous techniques, but there are also limitations. There is room for criticism that the user must determine the reference value, which is an arbitrary choice.

## Structure coefficients (obtained from CFA)

### How often is it used?

AMJ 7.4%, JAP 0%, ORM 0%

### How to obtain

A typical CFA is an independent clusters (IC) model, where each item loads on just one factor and the loading value with other factors is zero. In the case of a CFA model specification, the term loading invariably refers to the factor pattern coefficients because the factor structure coefficients are not model parameters that are estimated, but are something that can be calculated post-estimation.

While Stata does not provide structure coefficients directly, we explain how to obtain them. Factor structure coefficients are the matrix multiplication of factor pattern coefficients and factor correlations. That is, the (i, j) th element of the third matrix is obtained by multiplying the (i, k) element of the first matrix by the element (k, j) of the second matrix, and summing it over all k. These can be calculated conveniently using Stata’s matrix command to multiply the two matrices.

. matrix structureCoefficients = loadings \* factorCorrelations

. matrix list structureCoefficients

structureCoefficients[11,4]

latent: latent: latent: latent:

ACSI CUEX PERQ PERV

observed:acsi1 1.8528717 1.1348305 1.7730513 1.6205307

observed:acsi2 1.7447974 1.068638 1.6696327 1.5260083

observed:acsi3 1.459294 .89377547 1.3964286 1.2763057

observed:cuex1 .96846481 1.5812415 1.1043172 .83175186

observed:cuex2 1.0085848 1.6467467 1.1500651 .86620835

observed:cuex3 .63565171 1.0378476 .72481839 .54592019

observed:perq1 1.6372332 1.1948963 1.7109393 1.3166735

observed:perq2 1.4957863 1.0916646 1.5631246 1.202921

observed:perq3 .9357882 .68296309 .97791617 .75256692

observed:perv1 1.4373643 .86447115 1.2647318 1.6434441

observed:perv2 1.6229873 .97611003 1.4280608 1.8556805

Let's determine the cross-loading according to some rules discussed above. First, absolute comparison. Let's choose .4 as the cutoff point. There are cross-loadings in all items except CUEX3. Second, row comparison. There is no cross-loading on any item. Third, column comparison. Cross-loadings can be found in five items: ACSI2, ACSI3, CUEX3, PERQ2, and PERQ3.

## CICFA(.9): the proposed technique

### How often is it used?

AMJ 0%, JAP 0%, ORM 0%

There are no studies that formally proposed this technique.

### How to obtain

The method of obtaining the upper limit of factor correlation in R has been described above (i.e.,Table 42). The difference between the existing technique and CICFA (.9) is what this upper limit is compared to. The existing technique evaluates that there is a problem with discriminant validity if the upper limit is larger than 1. Our concern is not whether the correlation is exactly 1, but whether it is a sufficiently low number of 1, so the existing technique does not fit well with the definition of discriminant validity. We suggest using a number less than 1 as a cutoff. CICFA (.9) evaluates that there is a problem with discriminant validity if the upper limit is larger than .9. The upper limit of the correlation between ACSI-PERQ is .962, which does not pass this criterion.

### Problems / Limitations

The choice of a cutoff is inevitable for a dichotomous (i.e., yes or no) decision, but there is room for criticism that any cutoff is arbitrary. More research is needed as to which cutoff is best, and different cutoffs may be considered depending on the purpose of the study (e.g., CICFA(.85)).

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1. Because the example data originates from four scales intended to measure four different constructs, theory would suggest that four factors should be estimated. However, to simplify the example we decided to extract just two factors. [↑](#footnote-ref-1)
2. Because the example data originates from four scales intended to measure four different constructs, theory would suggest that four factors should be estimated. However, to simplify the example we decided to extract just two factors. [↑](#footnote-ref-2)
3. Because the example data originates from four scales intended to measure four different constructs, theory would suggest that four factors should be estimated. However, to simplify the example we decided to extract just two factors. [↑](#footnote-ref-3)